Geology 376-2, Spring 2004, Assignment 1

You are encouraged to work with others (and to ask me questions), but you should compose and submit your solutions independently. Please show your work.

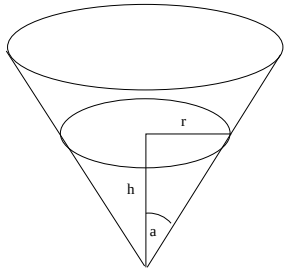
1. A conical water tank is being filled with pure gallium at a rate of 2 liters per second, as shown in the figure. The half-vertex-angle $a$ of the cone is $\pi/6$ (radians). Your job is to figure out how fast the level in the tank is rising. By the way, the volume of a cone is given by $v = \pi r^2 h/3$. Here's what I recommend:

A. Using $a = \pi/6$, express $r$ in terms of $h$, and thus $v$ in terms of $h$.

B. Invert the relationship in part A to get $h$ in terms of $v$.

C. Now you've got a function $h(v)$, where $v$ depends on $t$. Find $dh/dt$.

D. How fast is the water level (I mean, gallium level) rising when $t = 9$, assuming that the tank starting filling at time $t = 0$? Please give an exact answer, and a decimal approximation if you like.




2. In parts A, B, and C, compute the indicated antiderivative. Part D asks you to compute an improper definite integral. You are to enjoy the entire process.

A. $\int 2^x \, dx$

B. $\int x \ln x^2 \, dx$

C. $\int x \ln x \, dx$

D. $\int_0^1 x \ln x \, dx$

3. Recall that the Bessel function $J_0$ is defined by the power series

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}.$$

Your job is to show that $y = J_0(x)$ satisfies the differential equation

$$yx^2 + y'x + y''x^2 = 0.$$

Here are the steps that you will probably follow:

A. Compute $y'$ and $y''$ by differentiating $J_0(x)$ term-by-term.

B. Show that

$$y'x + y''x^2 = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{2^{2(n-1)}((n-1)!)^2}.$$

C. Let $m = n - 1$, and rewrite part B to show that $y'x + y''x^2 = -yx^2$.