Geology 376-2, Spring 2004, Assignment 3

You are encouraged to work with others (and to ask me questions), but you should compose and submit your solutions independently. Give exact answers, and show your work. Let me know if you find any errors.

1. I did not invent this problem, but I won't attribute it here, so that you don't look it up.

Suppose that we have two adjacent bodies, such as two rock units or two cells in an animal. Their volumes (measured in liters, cubic light years, teaspoons — I don't care) are changing over time, because material is flowing (1) from the first body into the second, and (2) from the second out into the ambient space. At any time, the rate of flow (1) is equal to three times the volume of the first body, and the rate of flow (2) equals two times the volume of the second body.

A. Let $v_1(t)$ and $v_2(t)$ represent the volumes of the two bodies. Describe the situation as a system of two differential equations involving v_1 and v_2.

B. Find the general solutions for v_1 and v_2.

C. Suppose that at time $t = 0$ the first volume is 40 and the second volume is 3. Use these initial conditions to determine the specific solutions for v_1 and v_2.

2. Solve the same problem again, with one change: The rate of flow from the first body into the second now equals just two times the volume of the first body.

This changes the behavior of the solutions qualitatively, because the coefficient matrix is no longer diagonalizable. To solve the system, solve the simpler equation, substitute your answer into the other equation, and solve that using one of the methods covered in class.