# Math 32-05/06, Fall 2005, Exam 2

Name:

I have adhered to the Duke Community Standard in completing this examination.

Signature:

Instructions: You have 50 minutes. Calculators are not allowed. Always **show all of your work**. Pictures are often helpful. Partial credit may be awarded. Give **simplified**, **exact** answers, and **draw a box** around them.

**1**. Differentiate the following functions.

A. $y(x) = \ln \cosh x$

B. $y(x) = \tan^{-1}(\sinh x)$

C. $y(x) = \int_1^{\cosh x} \frac{1}{t^2-1}\, dt$

D. $y(x) = (e^x)^{e^x}$

**2.** There are three antiderivatives below. One cannot be computed with our current methods; write "CANNOT COMPUTE" under it, and compute the other two antiderivatives.

A. $\int \frac{2x}{\sqrt{x^4-1}}\,dx$

B. $\int \frac{2}{\sqrt{x^4-1}}\,dx$

C. $\int \frac{2}{x\sqrt{x^4-1}}\,dx$

**3**. This exercise concerns the hyperbolic tangent function, $y = \tanh x$.

A. Show that this function is one-to-one (that is, invertible).

B. What are the domain and range of this function? Sketch a graph, including all intercepts and asymptotes. (Hint: In doing this, you may find it useful to compute some limits.)

C. Now let $y = \tanh^{-1} x$ be the inverse hyperbolic tangent. Prove that $\frac{dy}{dx} = \frac{1}{1-x^2}$.

**4**. A falling object accelerates down at 32 ft/s$^2$, ignoring drag (air resistance). If we take drag into account, then it yields an additional acceleration *up*, proportional to the square of the velocity. The constant of proportionality depends on mass, surface area, shape, etc. For simplicity, we will assume that this constant is 1/2; that is, the acceleration *down* due to drag is $-\frac{1}{2}$ times the square of the velocity.

A. Write a differential equation that describes how the velocity of the falling object is affected by gravity and drag.

B. Solve the differential equation for $v(t)$, assuming that the object begins at rest.

C. What is the terminal velocity, meaning the limit of $v(t)$ as $t$ goes to infinity? (Assume that the object keeps falling forever. Show your limit computation in detail.)

**5**. Compute the given limits.

A. $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\ln(1+x)}\right)$

B. $\lim_{x\to \pi/2^-} (\pi/2 - x)\tan x$

C. $\lim_{x\to 0} (1+ax)^{1/x}$