Math 32-05/06, Fall 2005, Exam 4

Name:

I have adhered to the Duke Community Standard in completing this examination.

Signature:

Instructions: You have 50 minutes. Calculators are not allowed. Always **show all of your work**. Pictures are often helpful. Partial credit may be awarded. Give **simplified**, **exact** answers, and **draw a box** around them. You may assume the following trigonometric identities:

$\cos^2 x + \sin^2 x = 1$
$1 + \tan^2 x = \sec^2 x$
$\cot^2 x + 1 = \csc^2 x$
$\sin(x + y) = \sin x \cos y + \cos x \sin y$
$\cos(x + y) = \cos x \cos y - \sin x \sin y$
$\sin 2x = 2 \sin x \cos x$
$\cos^2 x = (1 + \cos 2x)/2$
$\sin^2 x = (1 - \cos 2x)/2$
$\cosh^2 x - \sinh^2 x = 1$
$1 - \tanh^2 x = \mathrm{sech}^2\mathrm{x}$
$\coth^2 x - 1 = \mathrm{csch}^2\mathrm{x}$
$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$
$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$
$\sinh 2x = 2 \sinh x \cosh x$
$\cosh^2 x = (\cosh 2x + 1)/2$
$\sinh^2 x = (\cosh 2x - 1)/2$

1. In general, what does $\sum_{n=0}^{\infty} a_n = A$ mean? That is, how is the value of a series defined?

2. Find the interval of convergence of

$$\sum_{n=1}^{\infty} \frac{3^n x^n}{n^3}.$$

3. A. Write $\ln 3/2$ as a series.

B. How many terms are required to approximate $\ln 3/2$ with an error smaller than 10^{-2}?

C. What is the value of the approximation with that many terms?

D. What happens if you use the same method to compute $\ln 5/2$? Explain in detail.

4. For each series, determine whether it diverges, converges conditionally, or converges absolutely. Explain your answers.

A.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^n}{3^{n^2}}$$

B.

$$\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{\sqrt{n}}$$

5. Solve the differential equation $y' - xy = 0$ using the method of power series. If your series answer is a function that you recognize, then express the function in closed form as well.

6. Find the Taylor series for $f(x) = \ln(1 + x)$ centered at $a = 0$ explicitly, by taking successive derivatives and using Taylor's formula.

7. Use power series to compute $\int_0^1 \sin(x^2)\, dx$. You should leave your answer as a series, simplified as much as possible.