Name:

I have adhered to the Duke Community Standard.

Signature:

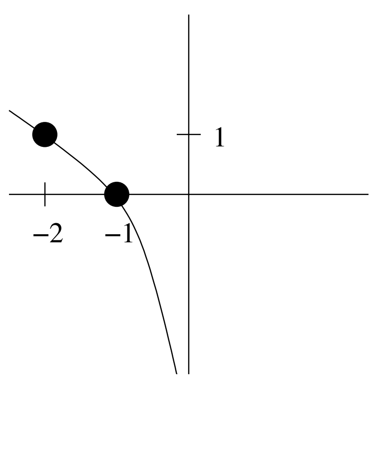
Math 31L 03-04 Fall 2006 Exam 1

Instructions: You have 70 minutes. You may use your TI-83 or equivalent calculator. Always show all of your work. Partial credit is often awarded. Pictures are often helpful. Give simplified answers, as exact as possible. Put a box around each answer. Ask questions if any problem is unclear. Good luck.

1. In each of the following, draw a graph $y = f(x)$ with the specified properties, or explain briefly why no such graph can exist.
A. $f(x)$, $f'(x)$, and $f''(x)$ are all negative for all x.

B. f is differentiable, $f(x)$ is always positive, and $f''(x)$ is negative for $x < 0$.

2. Find a function $y = f(x)$ whose graph could be the one depicted below.



3. Compute the derivatives of the following functions.
A. $f(x) = 3x^5 - 2x$

B. $f(x) = x^e - e^x$

C. $f(x) = 3^x \ln 3$

D. $f(x) = \frac{x^4+1}{ex^4-2x}$

E. $f(x) = 2^7$

4. Climate scientists in the German Alps have been studying Musterhorn Glacier B, which I just made up. This is a giant, rectangular block of ice. Let w denote its width, h its height, and ℓ its length, all in kilometers. Its volume is $v = wh\ell$. Its height and length have been decreasing over recent years due to climate change, as the chart below shows. Its width is a constant 1.31 km, because it is trapped in a steep valley between two mountains.

Year t	Height $h(t)$ (in km)	Length $\ell(t)$ (in km)
1980	0.82	2.15
1990	0.80	2.12
2000	0.79	2.10

A. Numerically estimate $h'(t)$ at $t = 1990$.

B. What does the quantity that you computed in Part A mean about the glacier? What are its units?

C. Numerically estimate $\ell'(t)$ at $t = 1990$.

D. Using your answers to Parts A and C, and the product rule, estimate the rate of change of the volume of the glacier in 1990.

E. What was the volume v of the glacier in 1990? In 2000? Using these, make another estimate the rate of change of the volume in 1990.

5. This problem deals with an unknown function $y = f(x)$. All we know is that $f'(x) = \frac{1}{2-x^2}$ and that $f(0) = 3$.
A. What is the linear approximation to $f(x)$ at $x = 0$?

B. Using the linear approximation of Part A, estimate $f(1)$.

C. Using your answer to Part B, find a linear approximation to $f(x)$ at $x = 1$.

D. Using your answer to Part C, estimate $f(2)$.

E. Using your results thus far, sketch an approximate graph of $y = f(x)$ for $0 \leq x \leq 2$.

F. The approximation goes bad somewhere between $x = 1$ and $x = 2$. Why?

6. Use the definition of the derivative to compute the derivative of the function $f(x) = \frac{1}{2x-3}$.

7. Find the following limits or explain briefly why they do not exist.

A. $\lim_{x\to 0} \frac{3x^2-4x+1}{-2x^2+2}$

B. $\lim_{x\to \infty} \frac{3x^2-4x+1}{-2x^2+2}$

C. $\lim_{x\to 1} \frac{3x^2-4x+1}{-2x^2+2}$