Math 103-03, Spring 2006, Sample Questions for Exam 2

WARNING: “Answers” are on the next page. Don’t look.

This document should not be regarded as a study guide or comprehensive review. Its purpose is simply to give examples of how an exam writer might transform abstract or mechanical book problems into slightly more applied or conceptual problems on exams.

My recommendation is that you seriously attempt all of the problems below, and then read the next page and reattempt any ones you couldn’t solve at first. Then, as you study more problems from the book, practice wrapping them in a similar veneer of conceptual difficulty; trade problems with your studying partners.

1. Joe Lagrange is trying to make money selling 50-gallon drums of olive oil over the Internet. By studying the relationship between cost and profit, he has determined that his *profit* on a drum depends on the *cost* (and only the cost) of the drum, in a complicated way that he still doesn’t quite understand. The cost of a drum is the sum of two terms: the *materials cost* and the *shipping cost*. What is the relationship between the following two quantities?

- the rate of change of profit with respect to materials cost
- the rate of change of profit with respect to shipping cost

2. I begin with a chunk of uranium shaped exactly like the ellipsoid $2x^2 + 2y^2 + z^2 = 18$. I then drill a hole of radius 2 straight through the chunk, along the z-axis. What volume of uranium do I remove from the chunk, in doing this?

3. A pendulum of length L meters oscillates with period $T = 2\pi\sqrt{L/g}$, where $g = 9.8$ (in m/s^2) is the acceleration due to gravity. I want to compute the period of a pendulum of length 2. Because I’m lazy, I use an approximate value of $g = 9$. Using differentials, estimate the error produced by this approximation.

4. Sophie Germain is taking a casual stroll in the Himalaya Mountains. At one point she stops for a snack, at an elevation of 20000 feet. At this point the ground rises to the east with a slope of 3 and rises to the south with a slope of 2. In what direction should she go, to go downhill as directly as possible?

5. Empty space is often not really empty; suppose that empty space in our solar system contains 3 particles of dust per cubic meter. The radius of the Earth is about $6 \cdot 10^6$ meters, and it orbits the sun at a radius of about $150 \cdot 10^9$ meters. As it orbits, it picks up all particles of space dust that happen to lie in its path. How many particles of dust does it accumulate in a year?

1. This is 13.7 #41.

2. This is 14.4 #32.

3. This is similar to 13.6 #39.

4. This is not a book problem. Suppose that the landscape is the graph $z = f(x, y)$ with the positive x-axis pointing east and the positive y-axis pointing north, as usual. Then at Sophie's location $f_x = 3$ and $f_y = -2$, so $\nabla f = \langle 3, -2 \rangle$. Now what direction is downhill?

5. The set of points lying in Earth's annual path forms a torus; we just want to find the volume of this torus (and then multiply by 3). This is 14.4 #35 with $a = 6 \cdot 10^6$ and $b - a = 150 \cdot 10^9$; you could also solve it using Pappus' first theorem (14.5).