A summary of the basic integral theorems of vector calculus.

Part One. $n = 2$**.**

Suppose

$$\mathbf{F} \;=\; P\mathbf{i} + Q\mathbf{j}$$

Green's Theorem. Suppose R is a bounded region in $\mathbf{R}^2$ with boundary C. Suppose $\mathbf{T}$ is a unit tangent vector to C which points in the counterclockwise direction on the outer part of C and in the clockwise direction on the inner part of C. Then

$$\int_C \mathbf{F} \bullet \mathbf{T}\, ds \;=\; \iint_R \nabla \times \mathbf{F}\, dA.$$

$\nabla \times \mathbf{F}$ here is, by definition, the scalar $Q_x - P_y$.
Remark. This may also be written

$$\int_C P\, dx \;+\; Q\, dy \;=\; \iint_R Q_x - P_y\, dxdy$$

where C is oriented as before.
The Divergence Theorem. Suppose R is a bounded region in $\mathbf{R}^2$ with boundary C. Suppose $\mathbf{n}$ is the outward pointing unit exterior normal to R along its boundary C. Then

$$\int_C \mathbf{F} \bullet \mathbf{n}\, ds \;=\; \iint_R \nabla \bullet \mathbf{F}\, dA.$$

Part Two. $n = 3$**.** Suppose

$$\mathbf{F} \;=\; P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}.$$

Stokes' Theorem. Suppose S is a surface in $\mathbf{R}^3$ with boundary C and unit normal $\mathbf{n}$. Suppose $\mathbf{T}$ is the unit tangent field along C such that $\mathbf{n} \times \mathbf{T}$ points into S. Then

$$\int_C \mathbf{F} \bullet \mathbf{T}\, ds \;=\; \iint_S (\nabla \times \mathbf{F}) \bullet \mathbf{n}\, dA.$$

Remark. This may also be written

$$\int_C P\, dx \;+\; Q\, dy \;+\; R\, dz \;=\; \iint_S (R_y - Q_z)\, dydz \;+\; (P_z - R_x)\, dzdx \;+\; (Q_x - P_y)\, dxdy$$

where S is oriented as before.
The Divergence Theorem. Suppose T is a bounded region in $\mathbf{R}^3$ with boundary S. Suppose $\mathbf{n}$ is the outward pointing unit exterior normal to T along its boundary S. Then

$$\iint_S \mathbf{F} \bullet \mathbf{n}\, dA \;=\; \iiint_T \nabla \bullet \mathbf{F}\, dV.$$