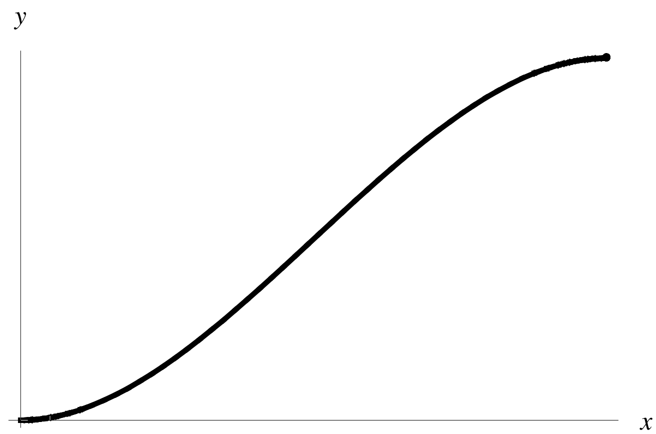

Figure 1: The graph of $y = P(x)$, as described in the problem.

To the writing portfolio reader: This assignment is taken from our textbook, *Calculus: Early Transcendentals*, 6th edition, by James Stewart. It appears there as Applied Project 3.4. The primary goal of this project was writing, not mathematics. Solutions were expected to be clear, self-explanatory, and in complete sentences. Students were encouraged to write as airline employees rather than math students; for example, in Problem 2 they would ideally prove that $\frac{6hv^2}{\ell^2} \leq k$, in a way that explains *why* such a condition is important to an airline. The audience was assumed to be skilled with algebra and familiar with calculus. Most students revised their solutions several times, based on feedback from the instructor.

An approach path for an aircraft landing is shown in the figure and satisfies the following conditions:

1. The cruising altitude is h when descent starts at a horizontal distance ℓ from touchdown at the origin.
2. The pilot must maintain a constant horizontal speed v throughout descent.
3. The absolute value of the vertical acceleration should not exceed a constant k (which is much less than the acceleration due to gravity).

1. Find a cubic polynomial $P(x) = ax^3 + bx^2 + cx + d$ that satisfies condition 1 by imposing suitable conditions on $P(x)$ and $P'(x)$ at the start of descent and at touchdown.

2. Use conditions 2 and 3 to show that

$$\frac{6hv^2}{\ell^2} \leq k.$$

3. Suppose that an airline decides not to allow vertical acceleration of a plane to exceed $k = 860$ mi/h^2. If the cruising altitude of a plane is $35,000$ ft and the speed is 300 mi/h, how far away from the airport should the pilot start descent?

4. Graph the approach path if the conditions stated in Problem 3 are satisfied.