

In order to help you understand the expectations for written work, I have written up my solutions to a few problems from early in the book. Some are short and others are long, to give you a good sampling. The key points to notice are:

- Each solution is written as a sequence of complete English sentences. When I feel that it is necessary, I throw in some words such as “because”, “since”, and “therefore” to help my reader follow my logic. I use the symbol “ $\Rightarrow$ ” to indicate that one equation or inequality implies another.
- Writing in complete sentences doesn’t mean that my solutions are long. In fact, I try to keep my solutions as succinct as possible.
- Each solution is self-explanatory. Sometimes I restate or paraphrase the problem, and sometimes I don’t, but I always make definitive statements that you can judge to be true or false without any extra information. You don’t need to read the statement of the problem in the book to follow what I did.
- Whenever I do a story problem, I explicitly define my notation, as in “Let  $r$  be the radius of the circle.” I also give units, if possible.

**1.5 #19.** If  $f(x) = 5^x$ , then

$$\frac{f(x+h) - f(x)}{h} = \frac{5^{x+h} - 5^x}{h} = \frac{5^x 5^h - 5^x}{h} = 5^x \left( \frac{5^h - 1}{h} \right).$$

**1.3 #53.** A stone is dropped into a lake, creating a circular ripple. Let  $r$  and  $A$  be the radius (in cm) and area (in  $\text{cm}^2$ ) of this circle. They depend on time  $t$  (in s). We are told that  $r$  increases at 60 cm/s.

A. Since  $r$  increases at a constant rate, it is linear in  $t$ . So  $r(t) = 60t + r_0$ , where  $r_0$  is the radius at  $t = 0$ . Assuming that  $t = 0$  is when the stone touches the water, we have  $r(t) = 60t$ .

B. Since  $A = \pi r^2$ ,

$$(A \circ r)(t) = A(r(t)) = \pi(60t)^2 = 3600\pi t^2.$$

This represents the area of the circle at time  $t$ .

**2.3 #38.** We wish to prove that  $\lim_{x \rightarrow 0^+} \sqrt{x} e^{\sin(\pi/x)} = 0$ . For any  $x$ , we have

$$\begin{aligned} -1 &\leq \sin(\pi/x) \leq 1 \\ \Rightarrow e^{-1} &\leq e^{\sin(\pi/x)} \leq e^1 \\ \Rightarrow e^{-1}\sqrt{x} &\leq \sqrt{x} e^{\sin(\pi/x)} \leq e^1 \sqrt{x}. \end{aligned}$$

Now, the functions on the left and right both limit to 0:

$$\begin{aligned}\lim_{x \rightarrow 0^+} e^{-1} \sqrt{x} &= e^{-1} \lim_{x \rightarrow 0^+} \sqrt{x} = e^{-1} \cdot 0 = 0, \\ \lim_{x \rightarrow 0^+} e^1 \sqrt{x} &= e^1 \lim_{x \rightarrow 0^+} \sqrt{x} = e^1 \cdot 0 = 0.\end{aligned}$$

Since both functions go to 0 and the function  $\sqrt{x}e^{\sin(\pi/x)}$  is trapped between them, the Squeeze Theorem says that it must also go to 0:

$$\lim_{x \rightarrow 0^+} \sqrt{x}e^{\sin(\pi/x)} = 0.$$

**3.1 #71.** We wish to find  $a$  and  $b$  such that the parabola  $y = ax^2 + bx$  has tangent line  $y = 3x - 2$  at  $(1, 1)$ . That is, we want the parabola to pass through  $(1, 1)$ , which means  $1 = a \cdot 1^2 + b \cdot 1 = a + b$ ; we also want the derivative  $dy/dx = 2ax + b$  to be 3, which means  $3 = 2a \cdot 1 + b = 2a + b$ . The simultaneous solution is  $a = 2, b = -1$ , so the parabola is  $y = 2x^2 - x$ .