Name:

I have adhered to the Duke Community Standard.

Signature:

Math 31L-02 Spring 2007 Exam 2

Instructions: You have 60 minutes. You may use your TI-83 or equivalent calculator. Always show all of your work. Partial credit is often awarded. Pictures are often helpful. Give simplified answers, as exact as possible. Put a box around each answer. Ask questions if any problem is unclear. Good luck.

1. Find $\lim_{x \to 0} \frac{\sin x}{x^{1/3}}$.

2. This question is about the function $y = \arccos x$.
A. What is the definition of this function?

B. Find its derivative. (Even if you have the answer memorized, you must show how it is arises.)

3. A conservation program is limiting the number of fish caught off the New Jersey coast. As a result, the fish have more time to mature and are larger when they're caught. Let $N(t)$ be the number of fish caught in year t of the program, and let $A(t)$ be the average mass of the fish caught in year t, in kg.

A. Let $M(t)$ be the total mass of fish caught. Give formulas for $M(t)$ and $M'(t)$.

B. In year 3 of the program, the average mass was 5 kg and increasing at a rate of 0.1 kg per year, and the number caught was $1,000,000$, decreasing at a rate of $100,000$ per year. What is $M'(3)$? Include units.

C. As less fish is caught, it becomes more valuable. Let $P(t) = \dfrac{1}{(M(t))^{3/2}}$ be the price in \$/kg, and let $R(t) = P(t)M(t)$ be the total revenue from selling fish in year t. Is $R(t)$ increasing or decreasing in year 3?

4. (In this problem, the units of time are seconds, the units of distance are meters, the units of mass are kilograms, and the units of force are kg m/s^2.) A particle of mass m is travelling inside a particle accelerator. Its velocity at time $t = 0$ is 4. At time t, it experiences a force of $F(t) = 1.1^t$.

A. What is the acceleration $a(t)$ of the particle?

B. What is its velocity, $v(t)$?

C. At time t, how far is it from where it started?

5. We have two reacting chemicals, A and B, with concentrations $[A] = [A](t)$ and $[B] = [B](t)$ governed by the differential equations

$$\begin{aligned}\frac{d[A]}{dt} &= -k_1[A] + k_2[B],\\ \frac{d[B]}{dt} &= k_1[A] - k_2[B].\end{aligned}$$

Let $A_0 = [A](0)$ and $B_0 = [B](0)$. Assume $[A]$ is decreasing at all times t.
A. Since $[A]$ is decreasing at $t = 0$, what can we conclude about the constants A_0, B_0, k_1, and/or k_2? Give your answer as an inequality.

B. Prove that $[B]$ is always increasing.

C. Is $\dfrac{d^2[B]}{dt^2}$ always positive, always negative, or neither? Prove it.

D. When is $[B]$ increasing the fastest?

6. Let a be a positive constant, and consider the curve $x^2 + y^2 = a^2$.

A. Compute $\frac{dy}{dx}$.

B. Compute $\frac{d^2y}{dx^2}$, as simplified as you can make it.

C. At which points (x, y) is the curve concave down? Concave up? (For full credit, you must explain based on your answer to Part B.)