

## An Equation Is A Statement

Here is a snippet of writing taken from a student's math homework.

$$18/2 = 9.$$

When we write such an equation, we are saying that the number  $18/2$  and the number  $9$  are actually the same number, written in two different ways. Similarly, when we write

$$\tan^2 x + 1 = \sec^2 x.$$

we are saying that the function  $\tan^2 x + 1$  and the function  $\sec^2 x$  are the same function — in other words, the number  $\tan^2 0 + 1$  is the same as the number  $\sec^2 0$ , the number  $\tan^2 1 + 1$  is the same as the number  $\sec^2 1$ , and in general, no matter what  $x$  is, the number  $\tan^2 x + 1$  is the same as the number  $\sec^2 x$ .

An equation is a statement — a sentence, complete with a subject and a verb, ending in a period, that makes a definite claim that two objects are equal. You can read the sentence aloud and make sense of it. If you can't, then it must not be correct.

For another example, consider the snippet

$$2(3.75 + 0.25) = 8 \cos 0.$$

You can rewrite this equation in plain text as

2, times the quantity 3.75 plus 0.25, equals 8 times the cosine of 0.

When you read it aloud, it should make sense, as a grammatical English sentence. The subject is “2 times the quantity 3.75 plus 0.25”. The verb is “equals”. The object is “8 times the cosine of 0”. The sentence ends in a period, like any other declarative English sentence.

(The subject and the object in this sentence are long, because they contain multiple prepositional subclauses. Mathematical sentences are often loaded with subclauses, which makes them difficult to read and potentially ambiguous. Before a lot of our mathematical notation was standardized in the 1700s, math really was written out in plain text, and reading it was painful; symbolic notation, although it can be intimidating to the uninitiated, is superior to plain text in its conciseness and precision.)

So far we've translated from symbols into plain text. Now let's try going the other way:

The sine of the quantity  $x$  plus  $y$  equals the sine of  $x$  times the cosine of  $y$ , plus the cosine of  $x$  times the sine of  $y$ .

This is a standard English sentence, just like those you write in your English or history papers. There is some math-specific jargon, but then again every discipline has jargon. The variables  $x$  and  $y$  might look odd in a sentence, but keep in mind that they are really just names like “Anna” or “Babatope” in ordinary English. The translation into symbols is

$$\sin(x + y) = \sin x \cos y + \cos x \sin y.$$

Read that sentence aloud; it should sound exactly as it does when written in plain text. Ultimately, the mathematical symbols are just a different way to spell the same old words.

Here is an example that is *not* an equation:

$$x^2 + 2x + 1.$$

You can tell that it's not an equation because there's no “=”. More importantly, it is not a sentence at all; when you read it aloud, you find that *there is no verb*. The reader cannot evaluate whether the writer is “correct” because the writer isn't making any claim at all.

In this course, you are expected to write all of your homework and exams in complete sentences. If a homework problem asks you to multiply out  $(x + 1)^2$ , do not just hand in “ $x^2 + 2x + 1.$ ” as above; instead make a complete statement:

$$(x + 1)^2 = x^2 + 2x + 1.$$

Finally, inequalities (which involve “<”, “≤”, “>”, or “≥” instead of “=”) are also statements, just as equations are. Read this one aloud, to convince yourself that it is a sensible English sentence:

$$\sqrt{x + 1} \geq \sqrt{x}.$$

A primary goal of this course is to communicate math in succinct, precise prose. Understanding that equations and inequalities are English statements is an important first step.

## Exercises

Read each expression aloud. Is it a sentence? If not, why not? If so, then is it true?

A.  $\log \sqrt{x^3} - 1.$

B.  $(t + 3)^2 = t^2 + 9$

Read each statement aloud. Then convert it from symbols to plain text (keeping numerals,  $x$ , and  $f$ , but no other symbols).

C.  $\sqrt{101} \geq \sin 0 + 15/1.5.$

D.  $f(x) = 3x - 2^x.$

Read each statement aloud. Then convert it from plain text to symbols.

E. 3 times the difference of 17 and 9 equals 24.

F. The square root of the quantity  $x$  squared, plus 2 times  $x$ , plus 1, equals plus or minus the quantity  $x$  plus 1.