You have 150 minutes.

You may use a calculator. You may use one 8.5×11-inch sheet of paper with notes written on it (on both sides) by you.

Show your work and explain your answers. Good work often earns partial credit; a correct answer with no explanation often earns little or no credit.

Good luck!

1. Compute the indicated derivatives.

A. $\dfrac{d}{dx}\left(5x^7 - 2x + 11\right)$

B. $\dfrac{d}{du}\dfrac{3^{\sin(\log_2 u)}}{\sqrt{u}}$

C. $\dfrac{d}{dt}\left(\sin t\right)^{\sin t}$

D. $\dfrac{d}{dx}\dfrac{d}{dx}\displaystyle\int_{\pi}^{x} 2u\sin(u^2)\,du$

2. Compute the indicated integrals using any method you like.

A. $\int_0^1 5x^7 - 2x + 11 \, dx$

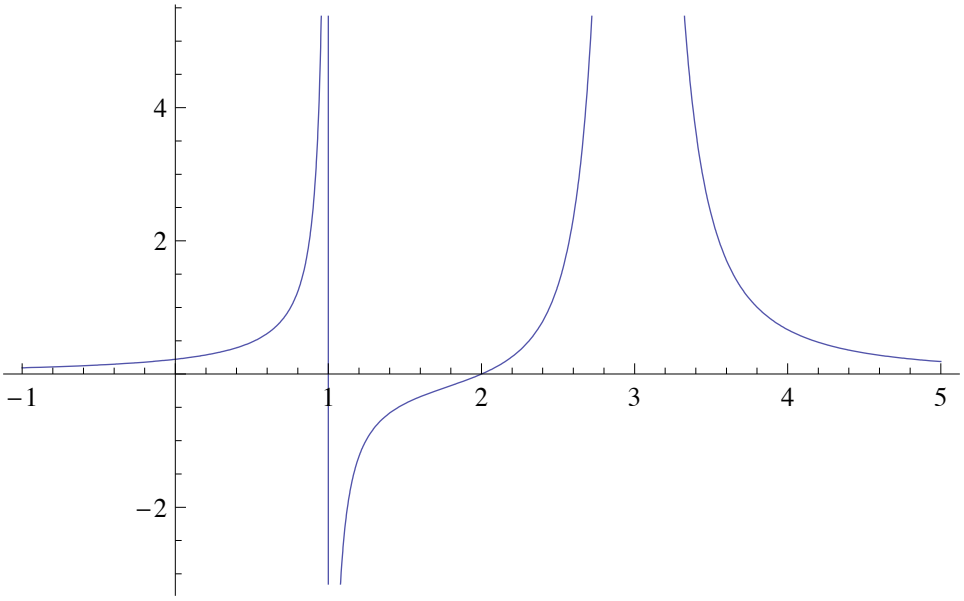
B. $\int_{-2}^2 \sqrt{4 - x^2} \, dx$ (hint: draw)

C. $\int_2^4 \frac{3t}{\sqrt{1 + 3t^2}} \, dt$

D. $\int_0^3 \frac{v + 1}{(v + 1)^2} \, dv$

3. Sketch the graph of a differentiable function $y = f(x)$, defined for all real numbers x, satisfying these criteria: (1) For $x < 0$, $f(x) < 0$. (2) For $x < 1$, $f'(x) < 0$. (3) For $x < 0$, $f''(x) < 0$. (4) For $x > 0$, $f''(x) > 0$.

4. Find a function whose graph might be the one pictured below. (There are vertical asymptotes at $x = 1$ and $x = 3$.) Box your answer.



5. Any object in, on, or near the Earth experiences a gravitational pull toward the center of the Earth. If the object is at distance x from the center, then the strength of the gravitational force is

$$f(x) = \begin{cases} \frac{c}{R^3}x & \text{if } 0 < x \leq R, \\ \frac{c}{x^2} & \text{if } x > R, \end{cases}$$

where $R \approx 6300$ km is the radius of the Earth and c is a constant depending on the Earth's mass and the object's mass. At which distance x is the force strongest?

6. Working from the *definition* of the integral, compute the area bounded by $x = 1$, $x = 5$, $y = 0$, and $y = 3x - 2$.

7. Police find a deceased person in an apartment in Dundas. They measure the body temperature to be 30 (degrees Celsius). An hour later they measure the body temperature and get 29. The apartment has been at a constant temperature of 20. Assuming that the person died at the "normal" temperature of 37 and subsequently obeys Newton's law of cooling, when did the person die? Show all steps.

8. A farmer wants to enclose a patch of land in a rectangular fence, and then divide it in half with another segment of fence running parallel to one of the rectangle's sides. The total of length of fence used will be 24 km. What is the largest area that can be enclosed in such a fashion?

9. Let R be the region of the plane enclosed by $y = 0$ and $y = -x(x-1)$. Let S be the solid obtained by rotating R about the x-axis. Draw a sketch of S, and find its volume.

10. Let f be any continuous function on the interval $[a, b]$. As thoroughly as possible, explain the distinction between these two objects:

$$\int f(x)\,dx \qquad \text{and} \qquad \int_a^b f(x)\,dx.$$