Here are some sample questions, like those that you might see on our in-class midterm exam. This document is not a contract; I am not promising that the exam will cover this material or that it will have this many questions. I have not carefully sampled, timed, and balanced the questions, as I do for a real in-class exam. You should regard this document as a *starting point* for studying; you will need to study/practice more problems than these. I regard these problems as varying from easy to medium-difficult.

Remember that no calculators or "cheat sheets" are allowed. You must show all work and explain all answers. Partial credit is often awarded. Exam grades are loosely curved. By this I do not mean that there are predetermined numbers of As, Bs, Cs to be awarded, but rather that there are no predetermined scores required for grades A, B, C. Typically the difficulty of my exams is such that the median score is around 65%. Scores above 95% are rare.

Let me reiterate that this is not an exam or a promise of what is going to be on the exam; it's just some questions I came up with, to help you study.

0. Compute the inverse (if it exists) of the matrix

$$\begin{bmatrix} 0 & 3 & 1 \\ 2 & 4 & -2 \\ -1 & 1 & 1 \end{bmatrix}.$$

1. Can the following objects exist? If so, give an example; if not, explain why not.

A. a 3×3 matrix of rank 1 with no 0s

B. a linear transformation $f : \mathbb{R}^2 \to \mathbb{R}^2$ representing translation by the vector $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

C. a linear transformation $f : \mathbb{R}^1 \to \mathbb{R}^3$ whose image is a plane

D. a linear transformation $f : \mathbb{R}^3 \to \mathbb{R}^1$ whose kernel is a plane

[Remark: These are essentially "true/false, but explain" questions. You can find many more true/false questions at the end of each chapter of our textbook.]

2. Let A be an $n \times n$ matrix such that $\ker(A^2) = \ker(A^3)$. Explain why it must be true that $\ker(A^3) = \ker(A^4)$. [Remark: I stole this from 3.1 #50.]

3. Let G be an undirected graph with n nodes $p_1, \ldots, p_n$. I am trying to come up with an algorithm (an unambiguous, reliable procedure) to determine, for any two nodes p_i and p_j, whether there exists a path from p_i to p_j. My friend Wichsinee suggests that I look at

$$A + A^2 + \cdots + A^{n-1},$$

where A is the adjacency matrix. Explain her idea.

4. Find the 3×3 matrix that represents the rotation of $\mathbb{R}^3$ that takes the positive x_1-axis to the positive x_2-axis, the positive x_2-axis to the positive x_3-axis, and the positive x_3-axis to the positive x_1-axis.

5. You're a hydrologist studying the groundwater under Carleton's campus. For simplicity, let's assume that campus is utterly flat. However, due to variations in the kinds of soil, clay, and rock under campus, the surface of the groundwater is not flat, as the surface of a lake is; the depth of the water's surface below ground varies from place to place. For example, here are some depths (in m) that you measure in an evenly spaced grid on the Bald Spot:

1.21	1.16	1.11	1.06
1.25	1.19	1.13	1.07
1.29	1.22	1.15	1.08
1.33	1.25	1.17	1.09

Now you want to understand the groundwater in the area of the Arena Theater. The problem is that you can't measure under the theater itself; you can only measure around it. The groundwater depths look like

1.22	0.91	0.75	0.83
1.43	a	b	0.91
1.29	c	d	0.93
1.23	1.17	1.09	1.04

where a, b, c, and d are unknown, because they're under the theater. Water diffusion theory tells you that *each depth measurement should equal the average of the four measurements adjacent to it.* Using this principle, give a system of linear equations that can be used to find a, b, c, and d. You do not have to solve them.

[Remark: This is similar to 1.1 #28, because the theory of heat diffusion is similar to the theory of water diffusion. To understand why, take Math 341.]

6. I have a set of n two-dimensional points $(x_1, y_1), \ldots, (x_n, y_n)$. I would like to find a polynomial

$$y = c_0 + c_1 x + c_2 x^2 + \cdots + c_d x^d$$

that passes through all n of the given points. What d should I pick? How do I find the polynomial using linear algebra?