Carleton College Math 111 01-02, Fall 2007, Exam 1

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October 7, 2007

You have 70 minutes.

You may not use any notes or calculator.

Always show your work and explain all of your answers. Good work often earns partial credit. A correct answer with no explanation often earns little or no credit.

Whenever possible, give a simplified answer and put a box around your answer so that it can be found easily.

If you have no idea how to solve a problem, or if you have forgotten a key formula that you think you need to know, you may ask me for a hint. The hint will cost you some points (to be decided unilaterally by me as I grade your paper), but it may help you earn more points overall.

Good luck.

1. Differentiate the following functions.

A. $y = f(x) = 5x^6 - 2x^3 + 3x + 1$.

B. $y = f(x) = 4^{\sin(\log(\sqrt{x}))}$.

2. We have seen a number of "limit laws" that help us manipulate limits. Is the following equation a valid limit law? That is, does it work for all functions f and g and all constants a?

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right).$$

3. The table below shows the mean (average) distances d of the planets from the sun (taking the unit of measurement to be the distance from the earth to the sun) and their periods T (time of revolution in years). We wish to model T as a function of d.

Planet	d	T	Planet	d	T
Mercury	0.387	0.241	Jupiter	5.203	11.861
Venus	0.723	0.615	Saturn	9.541	29.457
Earth	1.000	1.000	Uranus	19.190	84.008
Mars	1.523	1.881	Neptune	30.086	164.784

A. When I graph the data on a log-log plot, they seem to lie on a line of slope 1.499 and intercept 0.000431. What, then, is the function $T(d)$?

B. Kepler's Third Law of Planetary Motion says that "The square of the period of revolution of a planet is proportional to the cube of its mean distance from the sun." Does your model corroborate Kepler's Third Law?

4. You are standing on a cliff above a river. 10 meters above you is a bridge across the river. A bungie jumper is attached to the bridge by a bungie (a large rubber band). She jumps off the bridge and then bounces up and down for a while on the bungie. She finds this entertaining. Let time $t = 0$ be the moment she jumps; her altitude, relative to yours, is thereafter given by

$$y(t) = 10e^{-t}\cos(1.5t)$$

(where altitude y is in meters and time t is in seconds).

A. Find a formula for her velocity. What are its units?

B. Find a formula for her acceleration. What are its units?

C. Roughly how much time does she take to bounce up and down once?

D. At what altitude, relative to yours, does she eventually come to rest?

5. Differentiate $y = \sin(2x)$ from the definition of the derivative. In the course of doing so, you may assume the following two limits; any other limits you use must be explained thoroughly.

$$\lim_{h\to 0} \frac{\sin h}{h} = 1, \qquad \lim_{h\to 0} \frac{\cos h - 1}{h} = 0.$$

6. You are trying to make money on the stock market. You select a particular stock, download data about how its price has changed over time, and run the data through a statistics program to create a model that, you hope, can predict the future. The program says that the data are described well by

$$p(t) = 10 + \frac{t}{(t-1)^2(t-2)(t-3)^2},$$

where t is time (in years from now) and p is the stock price (in U.S. dollars).

A. What is the practical meaning of $p'(5)$? What are its units? (Do not compute it.)

B. In order to make a lot of money, you would like to purchase the stock when its price is low and later sell the stock when its price is high. According to the model $p(t)$ given, when are the best times for you to buy and sell?

C. Do you think that the $p(t)$ that your program found is a good model?

7. The sun is a big ball of gas not far from where you're sitting. For the sake of this problem, we will assume that it has uniform density (which therefore equals its mass divided by its volume). The sun's radius is $6.96 \cdot 10^8$ meters and its density is 1.41 tonnes per cubic meter. It loses $1.26 \cdot 10^{14}$ tonnes of mass per year. I want to figure out how fast its radius is shrinking.

A. First, identify all of the relevant quantities in the problem, and indicate how they relate to each other — that is, which quantities depend on which other ones?

B. Solve the problem — that is, compute how fast the radius is shrinking.

8. Prove that for any real number n the derivative of $y = x^n$ is $dy/dx = nx^{n-1}$. (Hint: Use logarithmic differentiation.)

9. Suppose that an environment of constant ambient temperature A contains a body of varying temperature $y = y(t)$. Then we have seen that

$$\frac{dy}{dt} = k(A - y),$$

where k is a positive constant. (You do not have to solve this differential equation here.)

A. If y starts out greater than A, then is dy/dt positive or negative? So what happens to y, according to the differential equation?

B. Similarly, if y starts out less than A, then what happens to it?

C. Similarly, if y starts out equal to A, then what happens to it?