Carleton College Math 111 01-02, Fall 2007, Exam 2

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You have 70 minutes.

You may not use any notes or calculator.

Always show your work and explain all of your answers. Good work often earns partial credit. A correct answer with no explanation often earns little or no credit.

Whenever possible, give a simplified answer and put a box around your answer so that it can be found easily.

If you have no idea how to solve a problem, or if you have forgotten a key formula that you think you need to know, you may ask me for a hint. The hint will cost you some points (to be decided unilaterally by me as I grade your paper), but it may help you earn more points overall.

Good luck.

1. Show, using any method you like, that $\int \log x \, dx = x \log x - x + C$.

2. Compute these. Remember to put boxes around your answers.

A. $\int \sqrt{t} \, dt$

B. $\int \sin(2\pi\theta) \, d\theta$

C. $\int x^2 \log(x^3) \, dx$

3. Suppose that $y = f(x)$ is a continuous function and a and b any two numbers. Thoroughly discuss the distinction between these two objects:

$$\int f(x)\,dx \qquad \text{and} \qquad \int_a^b f(x)\,dx.$$

4. Crows like to eat various mollusks. To crack open these hard-shelled creatures, they take them up in the air and drop them onto rocks. The longer the drop, the harder the impact and the more likely the mollusk is to crack open. On the other hand, flying to high altitudes requires a lot of energy. When trying to crack a mollusk, the crow naturally selects its dropping altitude in a way that minimizes the total energy required.

Let h be the crow's dropping altitude (in m). Assuming that the crow and mollusk together have mass 1kg, then each flight to altitude h requires the crow to expend energy h (in kg m^2/s^2). Also, it turns out that the number of drops required (on average) is

$$n(h) = 1 + \frac{16}{h - 1.2}.$$

A. Write a function $f(h)$ that describes the total energy required to crack a mollusk.

B. What value of h should the crow choose, to minimize its total energy expenditure?

5. Recall that the erf function is defined as

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt.$$

A. What is the domain of erf? That is, which values of x can be plugged into it?

B. It turns out that e^{-t^2} is difficult to antidifferentiate, so to compute erf we must resort to numerical approximations. In as much detail as possible, explain/show how one could estimate $\operatorname{erf}(x)$ for any given number x using Riemann sums (sums of areas of rectangles).

6. Recall from the previous page that $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$.

A. What is $\dfrac{d}{dx}\operatorname{erf}(x)$?

B. I want to solve $\operatorname{erf}(x) = 0.25$, but this is difficult, so I resign myself to an approximate solution. In as much detail as possible, explain/show how one could find an approximate solution using Newton's method.

7. My dentist keeps asking me about the following function for some reason. To get her off my back, please make a detailed graph of it, including intercepts, asymptotes, critical points, inflection points, the correct increasing/decreasing and concavity behavior, local maxima and minima, and anything else that you think is significant.

$$f(x) = x^{8/3} - x^{2/3}.$$

8. A space probe is shooting away from Earth. Its velocity at hour t is $27000 + 0.1\sqrt[3]{t}$ km/hr.

A. Write an integral that represents how far the probe travels in the first 24 hours.

B. Compute the integral.

9. This problem concerns linear approximation.

A. Find the linear approximation to $y = \cos x$ at $x = \pi/3$.

B. Using your linear approximation from Part A, estimate $\cos(1)$. Your answer may involve things like $\sqrt{\ }$ and π.

C. Using your linear approximation again, estimate $\cos(-1)$. Is this estimate good or not?