Carleton College Math 111, Winter 2008, Final Exam

You have 150 minutes.

You may not use any calculator. You may use one standard-size (8.5×11 inches) sheet of paper with notes written by you on both sides.

Show your work and explain your answers. Good work often earns partial credit. A correct answer with no explanation often earns little or no credit.

Whenever possible, give a simplified answer and put a box around your answer so that it can be found easily.

No hints will be given out on this exam. However, you should still feel free to ask clarifying questions.

Good luck.

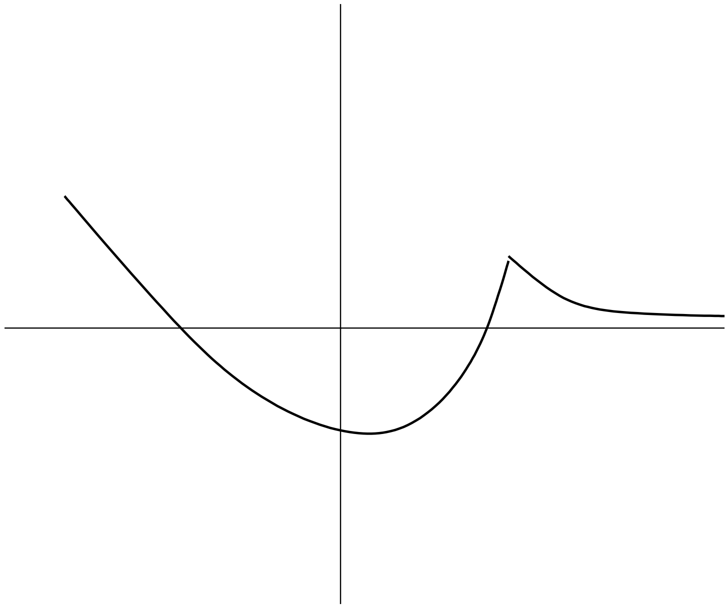
1. Compute.

A. $\dfrac{d}{du}17u^4 - 3u + 1$

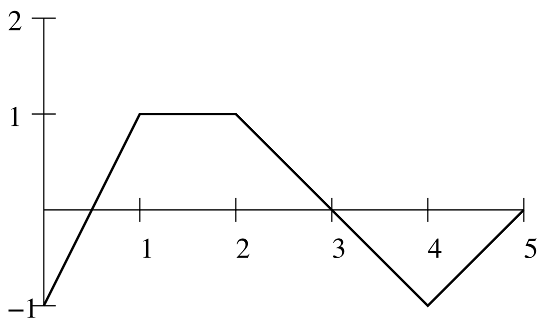
B. $\dfrac{d}{dt}\dfrac{\sin(5^t)}{\sqrt{\ln t}}$

C. $\dfrac{d}{dx}x^{(e^{2x}-1)}$

2. The graph of a function $y = f(x)$ is shown below. Sketch the graph of $y = f'(x)$ on top of it.



3. The following picture is a graph of $y = f'(x)$, not $y = f(x)$. In each part A, B, C, D there are two quantities. Between the two quantities, write a "$<$", "$>$", "$=$" to indicate that the quantity on the left is less than, greater than, or equal to the quantity on the right, or write "?" to indicate that the relationship is impossible to determine from the graph. You do not need to show any work.



A. $f(4) - f(1)$ $\quad$ 0

B. $f''(1/2)$ $\quad$ $f(3) - f(1)$

C. $f(2)$ $\quad$ 1

D. $f(5)$ $\quad$ $f(4)$

4. Compute.

A. $\int_{-1}^{1} 17u^4 - 3u + 1 \, du$

B. $\int_{1}^{2} \sin(e^t) \, e^t \, dt$

C. $\int_{0}^{1} x^3 \, (1 - x^2)^{3/2} \, dx$

5. Suppose that the engine in an automobile burns 1 liter of fuel per hour just to keep itself running (even when the automobile is not moving). When the automobile is going v km/h, the engine burns an *additional* $0.001v^2$ liters per hour to maintain that speed.

I'm about to take a 750 km road trip. I'd like to choose my speed v so that I use as little fuel as possible. By the way, I *never* exceed the speed limit, which is 90 km/h.

A. If I drive at constant speed v, then how many liters of fuel do I burn on my trip? Give your answer as a function $f(v)$. (Hint: How many hours long is the drive?)

B. How fast should I drive, to use as little fuel as possible on my trip?

6. Use the definition of the derivative to compute the derivative of the function $f(x) = \frac{1}{2x-3}$.

7. Find the following limits or show that they do not exist. Show all work.

A. $\lim_{x\to 0} \frac{3x^2-4x+1}{-2x^2+2}$

B. $\lim_{x\to \infty} \frac{3x^2-4x+1}{-2x^2+2}$

C. $\lim_{x\to 1} \frac{3x^2-4x+1}{-2x^2+2}$

8. Suppose that inside a volcano there is a chamber of molten hot *magma* (liquid rock). The magma chamber is a vertical box, 1 km wide in the east-west direction, 2 km long in the north-south direction, and extending from the surface down to a depth of 6 km.

As millennia pass, the magma cools into a block of solid rock. Due to a process known as *fractional crystallization*, the rock that forms at the bottom of the magma chamber is denser than the rock at the top. The density at depth y is

$$f(y) = 2.6e^{0.04y}$$

(in trillions of kg per km^3). It turns out that the total mass of the block is $\int_0^6 2f(y)\,dy$.

A. Explain in detail why this integral gives the total mass of the block. (Hint: Divide the rock into n slices of equal thickness, stacked vertically.)

B. Compute the total mass of the block.

9. You're at the beach with your brother Bart and sister Maggie. You want to know the temperature of the water. You pull a Krusty$^{\text{TM}}$-brand thermometer out of your pocket; it reads 30°C. You immediately plunge it into the water. After 1 minute it reads 20°C. After another minute, it reads 15°C. Find a function $y(t)$ for the temperature of the thermometer after t minutes, AND find the temperature of the water. Show all work, and circle both answers.

10. For all $x > 0$, define $f(x) = \int_0^{x^2} e^{-\sqrt{t}}\, dt$. Find all inflection points of $f(x)$.

11. In a coal mine, a conveyor belt is dropping chunks of coal onto a conical pile. As more and more coal is added, the pile continually shifts, so that it is always a cone with height equal to its radius. At 8:30 AM, the height of the pile is 6 m and growing at 2 m/hr. How fast is the volume of the coal pile changing at that time?

12. Suppose that $y = f(x)$ is a continuous function and a and b any two numbers. Thoroughly explain the distinction between these two objects:

$$\int_a^b f(x)\,dx \qquad \text{and} \qquad \int f(x)\,dx.$$

13. True or False (but explain thoroughly): Every continuous function has an antiderivative, and that antiderivative is continuous.

14. I'd like to find a zero of $f(x) = x^5 - 4x + 2$. In detail, describe an iterative process for solving this problem — approximately, at least.