This exam begins for you when you open (or peek inside) this packet. It ends at 5:00 PM on Tuesday 2009 May 19. Between those two times, you may work on it as much as you like. I recommend that you get started early and work often. The exam is open-book and open-note, which means, precisely:

(1) You may freely consult all of this course's material: the Kozen textbook, your class notes, your old homework and exam, and the materials on the course web site. If you missed a class and need to copy someone else's notes, do so before either of you begins the exam.

(2) You may assume all theorems discussed in class or in the assigned sections of the book. You do not have to prove or reprove them on this exam. On the other hand, you may not cite theorems that we have not studied. If you are unsure of whether you are allowed to cite a theorem, just ask.

(3) You may not consult any other papers, books, microfiche, film, video, audio recordings, Internet sites, etc. You may use a computer for these four purposes: viewing the course web site materials, editing and running Python programs relevant to the course, typing up your answers, and e-mailing with me. You may not share any materials with any other student.

(4) **You may not discuss the exam in any way (spoken, written, pantomime, semaphore, etc.) with anyone but me until everyone has handed in the exam — even if *you* finish earlier.** During the exam you will inevitably see your classmates around campus. Please refrain from asking even seemingly innocuous questions such as "Have you started the exam yet?" If a statement or question conveys any information, then it is not allowed; if it conveys no information, then you have no reason to make it.

During the exam you may ask me clarifying questions. If you believe that the statement of a problem is wrong, then you should certainly ask for clarification. I will not be giving out hints.

Your solutions should be thorough, self-explanatory, and polished (concise, neat, and well-written, employing complete sentences with punctuation). Always show enough work so that a classmate could follow your solutions. Do not show scratch work, false starts, circuitous reasoning, etc. If you cannot solve a problem, write a *brief* summary of the approaches you've tried. Submit your solutions in a single stapled packet, presented in the order they were assigned.

Partial credit is often awarded. Exam grades are loosely curved — by this I do not mean that there are predetermined numbers of As, Bs, Cs to be awarded, but rather that there are no predetermined scores required for grades A, B, C.

Good luck!

0. On our most recent assignment we implemented a calculator-like programming language interpreter. Write a detailed context-free grammar for this language. That is, your context-free grammar should describe syntactically valid commands to the interpreter and nothing else. (Remark: Usually one comes up with the grammar *before* implementing a language. In this case we didn't, for pedagogical reasons. The good news is that you can now experiment with the interpreter, to check whether your grammar is correct. If you have any doubt as to whether you completed the assignment correctly, then feel free to ask me for clarification.)

1. Prove that every regular language is also context-free. Do not go though the route of "every NFA is a PDA that doesn't use its stack"; instead, prove it by relating regular expressions to context-free grammars.

2. Consider the language A consisting of all strings of the form $x = y + z$, where $x, y, z \in \{0,1\}^*$ and $x = y + z$ is a correct equation of base-2 numbers. To clarify, A is a language over the alphabet $\{0, 1, +, =\}$. Is A context-free or not? If so, construct a context-free grammar; if not, prove it.

3. Give a formal definition of a *two-stack push-down automaton*; also, prove that the set of languages accepted by such automata is strictly larger than the set of context-free languages. (Remark: It may seem that there is no unique "correct" answer to the first part of this problem. However, there is an answer that is utterly natural and that facilitates the second part of the problem.)