

A. We've proven in class that $K(x)$ is not computable. So something must be wrong with the following argument. What's wrong?

We will build a Turing machine N that computes $K(x)$. Given an input x , N tests all strings y , in lexicographic order, to see whether they are descriptions of x . For each y , N first checks that y is of the form $\langle M, w \rangle$. If it is, then N runs M on w . N tests these strings y in parallel, in the usual way: one step on the first string, then two steps on the first string and one on the second, etc. As soon as N finds a string that describes x , N halts with the length of that string on its tape.

This works because there is a bound on the length of string that N must try. Let M be the Turing machine that immediately halts, and let $c = |\langle M, \rangle|$. Then, for any string x , $\langle M, x \rangle$ is a description of x of length $c + |x|$, and so $K(x) \leq c + |x|$. Thus N will find a description of x among the strings of length less than or equal to $c + |x|$.

B. Problem 6.23. (Hint: Mimic our proof that $K(x)$ is not computable.)

C. Prove that for any strings x and y , $K(xy) \leq c + 2 \log_2 K(x) + K(x) + K(y)$. What *exactly* is c ? (This problem is partially done in your book.)