

A. Show that the time complexity class P is closed under concatenation.

B. A language  $B$  is *NP-hard* if any language  $A \in \text{NP}$  can be reduced to  $B$  in deterministic polynomial time. A language  $B$  is *PSPACE-hard* if any language  $A \in \text{PSPACE}$  can be reduced to  $B$  in deterministic polynomial time. Show that any PSPACE-hard language is also NP-hard.

C. Let  $A$  be the set of all incompressible (according to our usual compression scheme) strings over  $\{0, 1\}$ . Is  $A$  recognizable?

D. In the USA, (simplified) postal addresses are formatted as in the two examples below. The first line is the *addressee*, an arbitrary string containing no carriage returns (ASCNCR). The second line is either a street address or a P.O. box. A *street address* is one or more digits followed by an ASCNCR. A *P.O. box* is “PO Box” followed by one or more digits. The third line consists of a city name (an ASCNCR with no commas), followed by a comma, followed by a two-letter state/territory code, followed by a ZIP code. The ZIP code is either five digits, or five digits followed by a dash and four digits. Write a regular expression for USA postal addresses as just described.

Michelle Obama  
1600 Pennsylvania Ave NW  
Washington, DC 20500

Minnesota Revenue  
PO Box 64054  
St Paul, MN 55164-0054

E. Let  $A = \{w\#t : w, t \in \{0, 1\}^*, w \text{ is a substring of } t\} \subseteq \{0, 1, \#\}^*$ . Use the pumping lemma to prove that  $A$  is not context-free.

F. Let  $A = \{\langle M \rangle : M \text{ is a Turing machine and } L(M) \text{ is finite}\}$ . Show that  $A$  is not recognizable, by describing a mapping reduction from  $\overline{\text{HALT}_{\text{TM}}}$  to  $A$ .