This exam consists of one cover page and five pages of problems.

You have 70 minutes. Notes, calculator, computer, etc. are not allowed.

When drawing an automaton, adopt the following convention to reduce clutter: If there is no transition out of a given state for a given input or stack symbol, then the machine rejects the input string, when it is in that state and sees that input or stack symbol.

You may cite without proof any theorem proved in class, in the assigned sections of the textbook, or in the assigned homework. You may not cite other results without proof.

If your solution to a problem is a straightforward application of a procedure discussed in class, then it may need no explanation. On the other hand, if it is novel, difficult to understand, or incomplete then it probably requires explanation. If you are unsure, then err on the side of explaining too much, or ask for clarification.

When there are multiple answers to a problem, it is understood that a simpler or more efficient answer may earn more credit.

Good luck.

A. In this problem, you will write a regular expression (RE) to match three-line USA postal addresses. Use textbook RE syntax; the only permitted operators are $\cup$, $\circ$, and $*$. You are encouraged to define auxiliary REs, to improve the clarity of your final answer.

Two examples are given below. One example is a street address, while the other is a post office box. One example has a ZIP+4 code, while the other has an old-fashioned ZIP code. Your final RE should handle all combinations of these possibilities. Feel free to ask for clarification.

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A. Write an RE Z that matches ZIP codes (including ZIP+4 codes).

B. Write an RE S that matches street addresses, such as the second line of the first example.

C. Write an RE P that matches P.O. boxes, such as the second line of the second example.

D. Write an RE, in terms of Z, S, and P, that matches three-line USA postal addresses.

B. Let A be a regular language and B a context-free language over the same Σ. Prove that $A \cap B$ is context-free. (Hint: Design a PDA for $A \cap B$? If so, then describe its parts explicitly.)

C. Let $A = \{1^n w : n \geq 0 \text{ and } w \text{ contains at most } n \text{ 1s}\} \subseteq \{0, 1\}^*$. If A is regular, then draw a DFA for it. If A is not regular, then prove so.

D. Let A be an infinite, regular language. Prove that there exist infinite, regular languages B and C (over the same alphabet as A) such that $B \cap C = \emptyset$ and $B \cup C = A$. (Hint: Pumping.)

E. Give a context-free grammar for $\{a^i b^j c^k : i = j \text{ or } j = k\} \subseteq \{a, b, c\}^*$.