This exam begins for you when you open (or peek inside) this packet. It ends at 8:30 AM on Monday 2013 May 20. Between those two times, you may work on it as much as you like. I recommend that you get started early and work often. The exam is open-book and open-note, which means, precisely:

(A) You may freely consult all of this course's material: the Sipser textbook, your class notes, your old homework and exam, and the materials on the course web site. If you missed a class and need to copy someone else's notes, do so before either of you begins the exam.

(B) You may assume all theorems and examples discussed in class or in the assigned sections of the book. You do not have to prove or reprove them on this exam. On the other hand, you may not cite unassigned problems or theorems that we have not studied. If you are unsure of whether you are allowed to cite something, just ask.

(C) You may not consult any other papers, books, microfiche, film, video, audio recordings, Internet sites, etc. You may use a computer for these three purposes only: viewing the course web site materials, typing up your answers, and e-mailing with me. You may not share any materials with any other student.

(D) **You may not discuss the exam in any way (spoken, written, pantomime, semaphore, etc.) with anyone but me until everyone has handed in the exam — even if *you* finish earlier.** During the exam you will inevitably see your classmates around campus. Please refrain from asking even seemingly innocuous questions such as "Have you started the exam yet?" If a statement or question conveys any information, then it is not allowed; if it conveys no information, then you have no reason to make it.

During the exam you may ask me clarifying questions. If you believe that the statement of a problem is wrong, then you should certainly ask for clarification. Check your e-mail occasionally throughout the exam period, in case I send out a correction or clarification.

Your solutions should be thorough, self-explanatory, and polished (concise, neat, and well-written, employing complete sentences with punctuation). Always show enough work so that a classmate could follow your solutions. Do not show scratch work, false starts, circuitous reasoning, etc. If you cannot solve a problem, write a *brief* summary of the approaches you've tried. Submit your solutions in a single stapled packet, presented in the order they were assigned.

Partial credit is often awarded. Exam grades are loosely curved — by this I do not mean that there are predetermined numbers of As, Bs, Cs to be awarded, but rather that there are no predetermined scores required for grades A, B, C.

Good luck!

A. Let $A$ be the set of all strings of the form $x = y \cdot z$, where $x$, $y$, and $z$ are binary numerals with no leading 0s, and the resulting equation is correct. That is, $A$ is a certain language over the alphabet $\{0, 1, =, \cdot\}$. For example, the string $10010 = 11 \cdot 110$ is in $A$. Prove that $A$ is not context-free.

Recall that, in a context-free grammar, each rule is of the form $A \to u$, where $A$ is a variable and $u$ is a string of variables and terminals. A *grammar* is much like a context-free grammar, except that rules of the form $v \to u$ are permitted, where $v$ and $u$ are both strings of variables and terminals. There is still a start variable $S$ and at least one rule of the form $S \to u$. To put it another way, a context-free grammar is a grammar in which, for every rule $v \to u$, $v$ consists of a single variable.

B. Let $G$ be a grammar and $L$ the set of strings generated by $G$. Prove that $L$ is recognizable, by describing in detail a Turing machine that recognizes it.

As you will soon see, this paragraph is filler material, put here simply to increase the amount of text on the page. It has no bearing on the exam whatsoever. When I was in college, one of my CS professors announced that he would be hosting a backyard barbecue for the CS majors. "This will be fun," many of us incorrectly assumed. When we arrived at the barbecue, there were just a few hot dogs and a few baked potatoes, for the 20 or so people who had shown up. Perhaps we were supposed to bring food? Anyway, we gave up on eating and started playing in our professor's pool. But the pool was very shallow, so we had to wade rather than swim. One student slipped and busted up his chin pretty badly.

Let $\mathcal{R}$ be the set of recognizable languages over $\Sigma = \{0, 1\}$. A *property of pairs of recognizable languages* is a function $P : \mathcal{R} \times \mathcal{R} \to \{\text{True}, \text{False}\}$. The function $P(L_1, L_2) = \text{True}$ is an example. The function $P(L_1, L_2) = \text{False}$ is another example. These two are called the *trivial* properties. A nontrivial example is the $P(L_1, L_2)$ defined by $L_1 \subseteq L_2$.

C. Prove that every nontrivial property of pairs of recognizable languages is undecidable.

Do not answer Problem D. I wrote it for this exam, and I'm leaving it on the paper as a practice problem to aid future studying, but it is not part of the exam. Answering it will not earn you extra credit, and not answering it will not cause you to lose points.

D. Let $A = \{a^n b^n c^n : n \geq 0\} \subseteq \{a, b, c\}^*$. Write a grammar for $A$.