Carleton College CS 202, Winter 2013, Quiz

This quiz is optional, but I strongly encourage you to take it, in one of two ways. The "more serious" way is to take the quiz as an out-of-class exam and submit it for grading on Wednesday 2013 January 23. The "less serious" way is to simply use this quiz as a set of sample problems, as you study for our first exam. Either way, your score on this quiz does not affect your course grade.

You have 60 minutes.

You may not use any notes, calculator, or computer.

Except on the TRUE/FALSE/PUNT questions, always show your work and explain all of your answers. Good work often earns partial credit. A correct answer with no explanation often earns little or no credit.

Good luck.

1. Each part of this problem is a true/false question, but there are three valid answers: TRUE, FALSE, and PUNT. If you answer PUNT, then you receive half credit. Otherwise, if you answer correctly then you receive full credit, and if you answer incorrectly then you receive no credit. No explanation is necessary on this problem. Do not just write T, F, or P; write the entire word.

A. About asymptotics: $\forall f\ \forall g\ \ (f = \mathcal{O}(g) \Leftrightarrow \exists N\ \forall n \geq N\ \ f(n) \leq g(n))$.

B. About propositional logic: For any proposition p, there exists a proposition q in conjunctive normal form such that $p \equiv q$.

C. About integers: $\forall a\ \forall b\ \forall c\ (a|bc \Rightarrow (a|b \vee a|c))$.

D. About people: Let $m(x, y)$ be "x is the mother of y". Then $\forall x\ \exists y\ m(x, y)$.

E. The truth table for the compound proposition $(p \Rightarrow q) \vee (r \Rightarrow \neg q)$ has exactly 6 true rows.

F. $\forall x\ \forall y\ p(x, y)$ is logically equivalent to $\forall y\ \forall x\ p(x, y)$ for all predicates $p(x, y)$ and all domains.

G. $\neg\forall x\ (p(x) \wedge q(x)) \equiv \exists x\ (\neg p(x) \wedge \neg q(x))$ for all predicates $p(x)$ and $q(x)$ and all domains.

H. Let p be the statement "Elvis is 6 years old" and q the statement "Elvis enjoys Disney World." Then the statement "Elvis enjoys Disney World only if Elvis is 6 years old" is $p \Rightarrow q$.

2. Prove that every odd integer is the difference of two squares. (Clearly distinguish between your scratch work and the proof that you want me to read.)

3. Consider the following two-player game. We begin with $n \geq 1$ squirrels on a submarine. The submarine is docked; standing on the dock, the two players take turns removing squirrels from the submarine. In her turn, a player must remove either 1 or 2 squirrels. The player that removes the last squirrel wins. Assume that neither player ever makes a mistake.

A. Prove that if n is divisible by 3 then the second player to move always wins.

B. Prove that if n is not divisible by 3 then the first player to move always wins.

4. For any integer $B \geq 2$, a B-ary tree is like a binary tree, except that each node may have up to B children, instead of just two children. In particular, a 2-ary tree is a binary tree.

A. Write a recursive definition of B-ary trees.

B. Prove that if a B-ary tree has n total nodes and height h, then $n \leq B^{h+1} - 1$. (For the sake of this problem, define the height of an empty tree to be -1.)