This assignment is in two parts. The first part is due at the start of class on Day 17. It will not be collected, but you are expected to complete these exercises, just to practice basic skills. If you feel that you need more practice, then do more problems or talk to me.

15.4 Exercises 11-12, 22, 39, 48

The second part is due on paper at the start of class on Day 19. Submit polished solutions, including all necessary work and no unnecessary work, in the order assigned.

A. In this problem, we assume that the Earth is a spherical ball of radius $R = 6371$ km. Measurements indicate that the density of the atmosphere drops off exponentially with altitude. To be precise, at an altitude of h km above the Earth's surface, the atmosphere has density $\delta(h) = ae^{-bh}$ kg/km^3, where $a = 1.225 \cdot 10^9$ and $b = 0.13$. Calculate the total mass of the atmosphere.

B. In quantum mechanics, the wave function for the 1s state of an electron in a hydrogen atom is $\psi(\rho) = (\pi a^3)^{-1/2} e^{-\rho/a}$, where $a = 5.3 \cdot 10^{-11}$ m is the Bohr radius. The probability of finding the electron in a region D of space is $\iiint_D |\psi(\rho)|^2 \, dV$. Compute the probability of finding the electron at a distance of R or less from the origin (nucleus).

C. Consider a body that occupies a region W of space and has density $\delta(x, y, z)$ (in kg/m^3). Assuming that each particle in the body began in the $z = 0$ plane, it took a certain amount of work to lift the particle to its current position. The total work, to raise all of the particles in the body from $z = 0$ to their current positions, was $\iiint_W \delta g z \, dV$, where $g = 9.8$ m/s^2. This concept can be used to calculate the energy needed to fill a water tower with water, for example. More interestingly, geologists use such concepts to understand how energy is distributed among geological processes such as mountain building and earthquakes.

The island of Hawaii is roughly a cone of height of 4200 m above sea level and radius 58000 m. Its density is roughly a constant 3000 kg/m^3. Based on these approximations, how much work did it take to raise Hawaii (the part above sea level) from sea level?

(By the way, this problem is very approximate. Hawaii is not really a cone, and its density is not constant. Most of Hawaii is under sea level, but we're ignoring all of that. The acceleration of Earth's gravitational field is not a constant 9.8 m/s^2 over such vast scales. In particular, the under-sea part of Hawaii dramatically alters the gravitational field experienced by the over-sea part.)