This work is due on paper at the start of class on Day 25. Submit polished solutions, including all necessary work and no unnecessary work, in the order assigned.

Let $\vec{P}$ be the point $(2, 0, 0)$ and $\vec{Q}$ the point $(-2, 0, 0)$. Let L be the line through $\vec{P}$ parallel to the y-axis and M the line through $\vec{Q}$ parallel to the z-axis.

A. Find a vector field $\vec{F}$, defined everywhere but L, such that curl $\vec{F} = \vec{0}$ but $\vec{F}$ is not a gradient. Also find a vector field $\vec{G}$, defined everywhere but M, such that curl $\vec{G} = \vec{0}$ but $\vec{G}$ is not a gradient. Where is $\vec{F} + \vec{G}$ defined? Does this vector field have curl zero? Is this vector field a gradient? Explain in detail.

B. Find a vector field $\vec{F}$, defined everywhere but $\vec{P}$, such that div $\vec{F} = 0$ but $\vec{F}$ is not a curl. Also find a vector field $\vec{G}$, defined everywhere but $\vec{Q}$, such that div $\vec{G} = 0$ but $\vec{G}$ is not a curl. What is $\operatorname{div}(\vec{F} + \vec{G})$?

Consider a circular pond of radius 4. At the edge of the pond is a cat, which can run at a speed of 4, but which cannot enter the water. The cat's goal is to catch a duck, which is at the center of the pond, and which can paddle at a speed of 1. The duck cannot take flight from the water, but it can take flight instantaneously and safely upon reaching the shore.

In these problems, we use parametrized curves to determine whether the duck can escape from the cat. The obvious idea is for the duck to paddle away from the cat, directly to the shore opposite the cat. But the duck takes 4 units of time to reach that point, and the cat takes only π units of time, so this is not a good strategy for the duck. A better idea is for the duck to continually adjust its course, as the cat comes around the pond to intercept it. The timing is delicate enough that fairly precise calculations are required.

C. Explain how the duck can move, so that the cat's resulting path is parametrized by $\vec{c}(t) = (4 \cos t, 4 \sin t)$, and the duck is as far away from the cat as possible at all times. What is the duck's position function $\vec{d}(t)$? Your answer for $\vec{d}(t)$ will probably involve an as-yet-unknown function $f(t)$.

D. What is $f(t)$? Why is this strategy ultimately inadequate for the duck?

E. Explain how to adjust the strategy for optimal duck performance. Does the duck escape or get caught? How much extra time does the duck or cat have?