This is a practice exam for our upcoming first exam. I recommend that you take this exam in a single hour-long period, in a distraction-free environment, to simulate the actual test experience. Although I have tried to make this practice exam resemble a real exam, you should not take this document as any kind of promise or contract about the content, length, or difficulty of our real exam. I design a real exam with more care than I have used in writing this practice exam.

You have 60 minutes.

No notes, books, calculators, computers, etc. are allowed.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

Perform as much algebraic simplification as you can. Do not bother to do non-trivial arithmetic. Simplify expressions such as $\sin 30°$ but not expressions such as $\sin 35°$. Mark your final answer clearly.

Good luck.

A. Let $\vec{v} = \langle 2, 1, -1 \rangle$ and $\vec{w} = \langle 1, 3, 3 \rangle$. Calculate the following.

$\vec{v} + \vec{w} =$

$4\vec{v} =$

$\vec{v} \cdot \vec{w} =$

$\vec{v} \times \vec{w} =$

B. Parametrize a circle of radius 10, in the plane $y = -2$, centered at the point $(4, -2, 3)$, such that the parametrization has constant speed 7.

C. Recall that in computer graphics a smooth surface is (usually) approximated as a set of flat triangles. This question is about a small piece of such a surface, near one of its vertices. Let $\vec{p}$, $\vec{q}$, $\vec{r}$, $\vec{s}$ be four three-dimensional vectors that define three triangles $(\vec{p}, \vec{q}, \vec{r})$, $(\vec{p}, \vec{r}, \vec{s})$, and $(\vec{p}, \vec{s}, \vec{q})$. In each triangle, I have listed the vertices in counter-clockwise order, when viewed from outside the surface. Notice that the three triangles meet at $\vec{p}$.

C1. Compute the unit outward-pointing normal vector for each triangle, in terms of $\vec{p}$, $\vec{q}$, $\vec{r}$, $\vec{s}$.

C2. From part C1 we have three normal vectors for the surface near $\vec{p}$. When used in lighting calculations, these can make the surface appear faceted and unsmooth. A better approach is to blend the three normal vectors together into a single unit normal vector, which is then used in lighting calculations near $\vec{p}$. How would you accomplish this blending?

D. Let $\vec{r} = (t, t^2, t^3)$ be the twisted cubic. Compute the unit normal $\vec{N}$ as a function of t.

E. The equation $r\cos(\theta - \pi/3) = 1$ describes a line in polar coordinates. Find polar equations for all lines parallel to this line.

F. Does $f(x, y, z) = \frac{3z^2}{x^2+y^2}$ have a limit at $(0, 0, 0)$? Explain thoroughly.

G1. Let $f(x, y) = e^{x^2+3y^2}$. Compute the linearization of f at $(x, y) = (2, 4)$.

G2. At that point, in which direction $(\Delta x, \Delta y)$ is f increasing most rapidly? How rapidly?