A. Prove that for any strings x and y, $K(xy) \leq c + 2\log_2 K(x) + K(x) + K(y)$. What *exactly* is c? (This problem is partially done in your book.)

B. We have proven (or will soon prove) in class that $K(x)$ is not computable. So something must be wrong with the following argument. What's wrong?

We will build a Turing machine N that computes $K(x)$. Given an input x, N tests all strings y, in lexicographic order, to see whether they are descriptions of x. For each y, N first checks that y is of the form $\langle M, w\rangle$. If it is, then N runs M on w. N tests these strings y in parallel, in the usual way: one step on the first string, then two steps on the first string and one on the second, etc. As soon as N finds a string that describes x, N halts with the length of that string on its tape.

This works because there is a bound on the length of string that N must try. Let M be the Turing machine that immediately halts, and let $c = |\langle M, \rangle|$. Then, for any string x, $\langle M, x\rangle$ is a description of x of length $c + |x|$, and so $K(x) \leq c + |x|$. Thus N will find a description of x among the strings of length less than or equal to $c + |x|$.

This last problem is harder. You probably won't be able to do it until after we prove that $K(x)$ is uncomputable. It's very good practice for this theory, however.

C. Problem 6.23. (Hint: Mimic our proof that $K(x)$ is not computable.)