Notes, book, etc. are not allowed. Unlike on our "practice exam", you must work alone.

Except where otherwise noted, you should always justify your answers. Correct answers with no justification may receive little credit. Incorrect or incomplete answers that display insight often receive partial credit.

If you feel that a problem is ambiguously worded, then explain your interpretation in your solution. Never interpret a problem in a way that renders it trivial.

You have 70 minutes. Good luck.

A. In each part of this problem, there are three valid answers: TRUE, FALSE, and PUNT. If you answer PUNT, then you get half credit. Otherwise, you get full credit for answering correctly and no credit for answering incorrectly. Justification is not required or considered in grading. Wherever it appears, M denotes a (deterministic, single-tape) Turing machine.

1. The union of any two decidable languages is recognizable.

2. If A is any language such that the complement $\bar{A}$ is finite, then A is decidable.

3. $\{\langle G, k \rangle : G$ is a directed graph with at least k edges$\}$ is decidable.

4. If M is in state q and sees tape symbol a, and later is again in state q and sees tape symbol a, then we can conclude that M is in an infinite loop and will never halt.

5. If N is any two-stack PDA, then $L(N)$ is decidable.

6. $\{\langle M \rangle : $ for all w, if M accepts w then M accepts the reverse of $w\}$ is decidable.

7. $\{\langle R, S \rangle : R$ and S are regular expressions, and $L(R) = L(S)\}$ is recognizable.

8. Let N be a nondeterministic Turing machine that, on input w, follows two branches: One branch rejects, and the other branch runs forever. Then N rejects w.

B1. What kind of transition function does a 3-tape Turing machine have? That is, describe the input and output types precisely. If you cannot give your answer in mathematical notation, then give it in plain English.

B2. Draw a (portion of a) 3-tape Turing machine that implements addition of numbers in binary. You may assume that the two summands are already loaded onto the first two tapes. Your job is to write the sum to the third tape and halt. On all three tapes, the bit order is the opposite of customary bit order: the ones bit is on the left, and the most significant bit is on the right.

C. Let A be the language consisting of all strings $\langle M, N, w \rangle$, where M and N are Turing machines, w is an input for M and N, and M accepts w if and only if N does not accept w. Prove that A is not recognizable.

D. Let $A = \{\langle D, M, C \rangle : C$ and D are DFAs, M is a TM, and $L(C) = L(D)/L(M)\}$. Prove that A is undecidable.