[These problems are intended to help you prepare for our first exam. However, they are not a promise of what the exam will look like. When I write an exam, I try to balance it carefully for difficulty (a mix of easy, moderate, and hard problems), content coverage, and length. I have not balanced these problems. They are like raw materials, from which I might craft an exam.]

No notes, books, calculators, computers, etc. are allowed.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

Perform as much algebraic simplification as you can. Do not bother to do non-trivial arithmetic. Mark your final answer clearly.

A1. What parameters does the binomial distribution have? If X is a binomially distributed random variable, then what is its set S of possible values, in terms of the parameters? What is its probability (density, mass) function? Compute its expectation.

A2. Do the same for the Bernoulli distribution.

A3. Do the same for the Poisson distribution.

A4. Do the same for the geometric distribution.

A5. Do the same for the uniform distribution.

A6. Are any of those five distributions special cases or limiting cases of the others? [I can think of three examples, off the top of my head.]

B. [This is Exercise 3.13.] In the card game known as bridge, the 52-card deck is dealt to four players, so that each player has 13 cards. Many bridge players believe that the most probable distribution of suits in a bridge hand is $(4, 3, 3, 3)$ (four cards of one suit and three cards of each of the other suits). Show that, in fact, the $(4, 4, 3, 2)$ distribution is more probable.

C. Prince Dotard, who is more confident than he is intelligent or wise, is planning to marry a beautiful princess and have 10 children. Based on the princesses available, the probability that his mate will have Zweibel's disease is $1/4$. If she does, then there is a small probability p that each of the children will also have the disease. What is the expected number of children with Zweibel's disease?

The negative binomial distribution is denoted $\mathrm{NegBin}(r, p)$ for parameters $r = 1, 2, 3, \ldots$ and $0 \leq p \leq 1$. A random variable $X \sim \mathrm{NegBin}(r, p)$ counts the number of Bernoulli trials (independent with probability p) needed to succeed r times.

D1. Find the probability (density, mass) function $P(X = k) = \ldots$ by considering all of the ways to have exactly r successes in exactly k trials and not before the kth trial. [If you need to glance at the answer, it's at the top of page 185 in our textbook.]

D2. When $r = 1$, the negative binomial distribution reduces to which other distribution that we've studied? Explain both conceptually and algebraically.

E. I'm going to describe a modification of the game called craps. As the player, you roll two dice and add them. If the sum is 7 or 11, then you immediately win \$1. If the sum is 2, 3, or 12, then you immediately lose (you win \$0). If the sum is any other number, then that number is called your *point*. In that case, you roll repeatedly until you get either 7, in which case you lose, or your point, in which case you win \$2. How much do you expect to win in a game?