You have 70 minutes.

No notes, books, calculators, computers, etc. are allowed.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

You may cite without proof any result discussed in class, the assigned textbook sections, or the assigned homework.

Perform as much algebraic simplification as you can. Do not bother to do non-trivial arithmetic unless it is specifically requested. Mark your final answer clearly.

"log" denotes the natural logarithm — base $e \approx 2.718$. Here are some miscellaneous facts (which you may quote without proof):

$$\begin{aligned}
\sum_{i=1}^{n} i &= \frac{n(n+1)}{2}, \\
\sum_{i=1}^{n} i^2 &= \frac{n(n+1)(2n+1)}{6}, \\
\sum_{i=1}^{n} i^3 &= \left(\sum_{i=1}^{n} i\right)^2.
\end{aligned}$$

Good luck. :)

There are no problems on this page. Use it for scratch work, if you like. Clearly mark anything that should be considered while grading.

A1. Let X be a random variable with density $f(x) = cx^{-2}$ on $[1, \infty)$. Find c.

A2. Let $Y = -\log X$. Find the probability density function of Y, including its domain.

B1. Let X and Y be discrete random variables. Under which conditions do $X + Y$ and $X - Y$ have covariance 0?

B2. How can one determine whether $X + Y$, $X - Y$ are independent? What must be checked?

C. An information technology specialist is planning a web site for a client. If she makes the site rich and sophisticated, then it will require a lot of computation per visitor. So, if visitors arrive frequently, then she will need a powerful computer to run the site, perhaps exceeding her client's budget. To help her plan the computation per visitor, find the mean and variance of the time between visitors, assuming that visitors arrive at a rate of 20 per minute, typically. Explain in English how knowing these quantities could be useful to her.

D1. An ecologist, who is studying predator-prey relationships, finds himself interested in distributions of the form $P(X = x, Y = y) = cx(x + y)$ for x in $\{1, \ldots, m\}$ and y in $\{1, \ldots, n\}$. Compute the marginal distributions of X and Y in terms of the constants c, m, n.

D2. Compute $P(X = x|Y = y)$.