You are assigned a few of these problems, to complete and turn in in 45 minutes. You may work with your assigned partner(s). You may bounce ideas off me. You may consult your class notes. You may not consult your textbook, the Internet, or any other source.

A. What is so unusual or notable about the vector field $\vec{F} = \vec{x}/|\vec{x}|^3$?

B. You are in charge of monitoring a large fleet of small space probes scattered throughout the solar system, all orbiting the sun in various ways. A few days after launch, you discover that their coolant levels are lower than they should be, by varying amounts, but holding steady. Perhaps coolant was lost during launch? This is a problem, because keeping a spacecraft from overheating is difficult, as heat cannot conduct away from the spacecraft's body in a vacuum. You want to figure out how high the temperature could get in your fleet of probes.

The temperature f of a probe depends on time, its position relative to the sun, and its coolant supply. What equations hold at the critical points of f, regarded as a function of time t and coolant c?

C. Let D be a 2D region, such that its boundary curve $C = \partial D$ is simple, closed, smooth, and connected. Let $\vec{n}$ be the outward-pointing unit normal vector along C. If $\vec{c}(s) = (x(s), y(s))$ is the counterclockwise arc length parametrization of C, then $\vec{n} = \langle y'(s), -x'(s) \rangle$. At any point of C, a smooth function $f(x,y)$ has a directional derivative $D_{\vec{n}}f$ in the direction of $\vec{n}$. Prove that

$$\int_C D_{\vec{n}}f \, ds = \iint_D \Delta f \, dA,$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

D. Consider a particle of electric charge q_1 situated at the origin, and a second particle of charge q_2 at position $\vec{x} = (x, y, z)$. Coulomb's law says that the electric force on the second particle is $\vec{F} = -\nabla V$, where $V(\vec{x}) = kq_1q_2/|\vec{x}|$ and $k > 0$ is a constant. Compute the work performed by $\vec{F}$ while the second particle moves from $\vec{P} = (1, 0, 2)$ to $\vec{Q} = (e^3, 1, 1 + e)$ along the curve $\vec{c}(t) = (e^{3t^2}, \sqrt[3]{t}, 1 + e^{t^3})$ for $0 \le t \le 1$. Also, assuming that the particle begins at rest at $\vec{P}$, and the sum of the kinetic and potential energy of the particle is constant through time, what is the particle's speed when it reaches $\vec{Q}$?

E. Let W be the region in the first octant ($x \ge 0$, $y \ge 0$, $z \ge 0$) bounded by the paraboloid $x^2 + y^2 = z - 2$ and the planes $x + y + z = 1$ and $x + y = 1$. Sketch W and compute its volume.

F. Let S be the sphere of radius R centered at (a, b, c). Use Lagrange multipliers to find the point on S that is closest to the origin.