You have 150 minutes.

No notes, books, calculators, computers, etc. are allowed.

You may cite without proof any result stated in class, in homework, or in the assigned sections of the textbook. If you are unsure of whether you can cite a result, then ask for clarification.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

Perform as much algebraic simplification as you can. Do not bother to do non-trivial arithmetic. Simplify expressions such as $\sin 30°$ but not expressions such as $\sin 35°$. Mark your final answer clearly.

Good luck.

A. Find a unit vector perpendicular to the plane through $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 3)$.

B. Parametrize the ellipse $(x/2)^2 + (y/3)^2 = 1$. Compute the unit normal $\vec{N}$ at each point.

C. Remember integration by parts? It's a theorem about integrals of scalar functions on closed intervals $[a, b]$ in the real line:

$$\int_a^b u \frac{dv}{dx}\, dx = [uv]_a^b - \int_a^b \frac{du}{dx} v\, dx.$$

Invent and prove a new kind of integration by parts, for triple integrals of scalar functions.

D1. Prove that curl (curl $\vec{F}$) $= \nabla(\text{div}\vec{F}) - \Delta\vec{F}$ (where $\Delta\vec{F}$ is defined component-wise).

D2. Maxwell discovered that an electric field $\vec{E}$ and its associated magnetic field $\vec{B}$ obey

$$\text{div}\vec{E} = 0, \quad \text{div}\vec{B} = 0, \quad \text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \text{curl } \vec{B} = k\frac{\partial \vec{E}}{\partial t},$$

where k is a constant. From Maxwell's equations, derive the wave equation $\Delta\vec{E} = k\dfrac{\partial}{\partial t}\dfrac{\partial \vec{E}}{\partial t}$.

E. Let α and β be two angles in a triangle, so that $\gamma = \pi - \alpha - \beta$ is the third angle. Find the values of α and β that maximize $\sin \alpha + \sin \beta + \sin \gamma$.

F. Do ONE of the parts below. Cross out the other part. Doing both earns no extra credit.

F1. Convert $f(z) = z^4 + c$ into a 2D vector field (as you did for $f(z) = z^3 + c$ on our homework).

F2. Sketch the Mandelbrot fractal that results from iterating the complex function $f(z) = z + c$. Also write a couple of sentences, to explain the important features of your sketch.

G. Let W be the 3D region bounded by the parabolic cylinder $y = x^2$ and the planes $x = z$, $x = y$, and $z = 0$. Sketch W and compute the integral of $x + 2y$ over W.

H. In the Cauchy momentum equation $\frac{D\vec{v}}{Dt} = -\frac{1}{\rho}\nabla p + \nabla \cdot T$, the pressure term is actually redundant. Explain why. In other words, show that the equation is equivalent to $\frac{D\vec{v}}{Dt} = \nabla \cdot U$, where U is what, exactly?

I. Let S be the portion of the ellipsoid $(x/4)^2 + (y/3)^2 + (z/2)^2 = 1$ where $x, y, z \leq 0$. Orient S so that it has upward-pointing normals. Compute the flux of $\vec{F} = \langle 0, 0, z \rangle$ across S.