

Solve Problems A-C on paper. Solve Problems D-F in a single Python file `day04.py`. Your code should include comments to explain any obscure or tricky bits. It should also include demonstration code. Hand in `day04.py` electronically, by dropping the file in your hand-in folder on the COURSES file server.

A. Working over  $\Sigma = \{a, b, c, d\}$ , draw an NFA that recognizes the same language as the regular expression

$$(ad \cup b \cup c)^*(dda)^* \cup ac.$$

B. Textbook regular expression syntax leaves out the complementation operation, even though regular languages are closed under complementation. Devise an algorithm for implementing complementation using the other regular expression operators. That is, your algorithm should begin with a regular expression  $R$ , and it should produce a new regular expression  $S$ , such that  $L(S) = \Sigma^* - L(R)$ . Illustrate your algorithm using the example of  $R = a \cup b$  over the alphabet  $\Sigma = \{a, b, c\}$ .

C. Hamming distance is an important concept in error-correcting codes. The *Hamming distance*  $H(w, x)$  between two bit strings  $w$  and  $x$  is defined as follows. If  $|w| \neq |x|$ , then  $H(w, x) = \infty$ . If  $|w| = |x|$ , then  $H(w, x)$  is the number of bits in which  $w$  and  $x$  differ. For example,  $H(00010, 10111) = 3$ . For any set  $A$  of bit strings, define  $N_2(A)$  to be the set of bit strings within Hamming distance 2 of  $A$ :

$$N_2(A) = \{w : \exists x \in A \text{ such that } H(w, x) \leq 2\}.$$

Prove that if  $A \subseteq \{0, 1\}^*$  is regular, then so is  $N_2(A)$ . (Hint: If  $A = L(M)$ , where  $M$  has states  $Q$ , then construct an NFA with states  $Q \times \{0, 1, 2\}$ . Use the extra state information to track how many “errors” have occurred thus far.)

D. Come up with a Python regular expression that describes mis-capitalized words. For the sake of this problem, a word is said to be mis-capitalized if it consists of two or more letters and any letter after the first one is upper-case. You may assume that only alphabetical characters appear in words. For example, when I feed your regular expression to

```
re.findall(yourregex, 'This is okay tHis IS nOT Okay.')
```

I should get a result of `['tHis', 'IS', 'nOT']`.

An e-mail address such as `sosupersnake@carleton.edu` consists of a local part, `sosupersnake`, and a hostname, `carleton.edu`. The local part is a string made of one or more characters from this set: upper- and lower-case English letters, the digits 0 through 9, the characters `!`, `#`, `$`, `%`,

&, ', \*, +, -, /, =, ?, ^, \_, ' , {, |, }, ~, and the period . . The hostname is a string made of one or more characters from this set: lower-case English letters, the digits 0 through 9, the period ., and the hyphen -. Neither the local part nor the hostname can begin or end in a period. The local part and the hostname are separated by a single @. (There are a few more rules to real e-mail addresses, but this is good enough for our purposes.)

E. Design a Python regular expression that matches e-mail addresses as just specified.

F. I like to write my dates in the format `yyyy/mm/dd`, but sometimes I accidentally write `mm/dd/yyyy` because that's how I was raised. Write a Python function `fixDates` that takes a string as input, uses a regular expression to fix all dates in the string to my liking, and outputs the fixed string. You may assume that all years are four-digit — I'm Y2K-compliant — but remember that months and days can be one- or two-digit. (Hint: You need to use substitutions and multiple groups.)