Let $f : \mathbb{N} \to \mathbb{N}$ be a function that grows without bound. That is, $\lim_{n\to\infty} f(n) = \infty$. In this problem, you will prove that $\mathcal{O}(2^{f(n)})$ is a proper subset of $2^{\mathcal{O}(f(n))}$.

A. Give rigorous definitions of $\mathcal{O}(2^{f(n)})$ and $2^{\mathcal{O}(f(n))}$. Prove that if g is $\mathcal{O}(2^{f(n)})$ then g is $2^{\mathcal{O}(f(n))}$. Also, prove that there is a g in $2^{\mathcal{O}(f(n))}$ that is not in $\mathcal{O}(2^{f(n)})$.

Earlier in our course, we described a Turing machine for testing whether a given directed graph was in fact a connected undirected graph.

B. What is the time complexity of that Turing machine? In addition to stating your answer in terms of the input size n, also state your answer in terms of the number m of nodes in the graph.