Here are some problems to help you study for Exam B. Try to complete them in 70 minutes, without using any book, notes, etc. Feel free to work on them with a partner.

These problems are like an exam, in that I have tried to make them cover a range of material, at a range of difficulty levels, in a way that is completable by a well-prepared student in 70 minutes. These problems are not a promise or contract about what our exam will look like. Our course web site offers actual exams from earlier terms, with solutions and quartiles. However, the number of exams, and whether they are take-home or not, varies from term to term.

A. In each part of this problem, there are three valid answers: TRUE, FALSE, and PUNT. If you answer PUNT, then you get half credit. Otherwise, you get full credit for answering correctly and no credit for answering incorrectly. Justification is not required or considered in grading. Unless otherwise noted, M denotes a (deterministic, single-tape) Turing machine.

1. $\{\langle M\rangle : M$ has 100 or more states$\}$ is decidable.
2. For every regular language A, the complement $\bar{A}$ is recognizable.
3. $\{\langle M\rangle :$ for all w, if M accepts w then M accepts $ww\}$ is decidable.
4. The intersection of any two context-free languages is also context-free.
5. A CFG in Chomsky normal form derives a string of length n in exactly $n+1$ steps.
6. If M is in state q and sees tape symbol a, and later is again in state q and sees tape symbol a, then we can conclude that M is in an infinite loop and will never halt.
7. Three-stack PDAs are equivalent in power to two-stack PDAs.
8. If A is any language such that $\bar{A}$ is finite, then A is decidable.

B. Prove that any context-free language is decidable. (We've proved this fact in class, but you cannot simply cite it from class. Your answer should demonstrate insight.)

Problem C is in two parts. Both parts concern two-tape non-deterministic Turing machines (2NTMs). We have never studied 2NTMs, but we have studied several related concepts.

C1. What kind of transition function should a 2NTM have? That is, describe the input and output types precisely. If you cannot give your answer in mathematical notation, then give it in English. But your answer should still be precise — as precise as the specification for a function or method in a well-documented computer program.

C2. Explain in detail how for any 2NTM N there is an equivalent two-tape deterministic Turing machine M. That is, M should accept, reject, and loop on inputs exactly when N accepts, rejects, or loops on them.

D. Let $A = \{\langle D, M, C\rangle : C$ and D are DFAs, M is a TM, and $L(C) = L(D)/L(M)\}$. Prove that A is not decidable. (Hint: You may cite without proof anything proved in our homework.)