

This exam begins for you when you open (or peek inside) this packet. It ends at 9:50 AM on Wednesday 2013 November 20. Between those two times, you may work on the exam as much as you like. Although I do not intend the exam to require more than a few hours, you should get started early, in case you want to spend more time. The exam is open-book and open-note:

- You may use all of this course’s materials: the Sipser textbook, your class notes, your old homework, and the materials on our course web site. If you missed a class and want to get some other student’s notes, then do so before either of you begins the exam. You may not share any materials with any other person while you are taking the exam.
- You may cite theorems and examples from class, and from the assigned sections of our textbook. You do not have to reprove them. On the other hand, you may not cite results that we have not studied.
- You may not consult any other books, papers, Internet sites, etc. You may use a computer for viewing the course web site, typing up your answers, and e-mailing with me. If you want to use a computer for other purposes, then confirm with me first.
- You may not discuss the exam in any way — spoken, written, etc. — with anyone but me, until everyone has handed in the exam. During the exam period you will inevitably see your classmates around campus. Refrain from asking even seemingly innocuous questions such as “Have you started the exam yet?” If a statement or question conveys any information about the exam, then it is not allowed. If it conveys no information, then you have no reason to make it.

Feel free to ask clarifying questions in person or over e-mail. You should certainly ask for clarification if you believe that a problem is mis-stated. Check your e-mail occasionally, in case I send out a correction.

Your solutions should be thorough, self-explanatory, neat, concise, and polished. You might want to work on scratch paper, and then recopy your solutions. Alternatively, you might want to type your solutions. Always show enough work and justification so that a typical classmate could understand your solutions. If you cannot solve a problem, then write a brief summary of the approaches you’ve tried. Partial credit is often awarded. Present your solutions in the order assigned, in a single stapled packet.

Good luck.

A. Is *PSPACE* closed under concatenation? If so, prove it. If not, find a counterexample.

B. In an undirected graph, a *stranger set* is a set of nodes, no two of which share an edge. For example, if the graph is a social network, then a stranger set is a set of people who don't know each other — hence the name. Let

$$A = \{\langle G, k \rangle : G \text{ is an undirected graph containing a stranger set of size } k\}.$$

Prove that A is *NP*-complete. (Hint: Reduce *3SAT* to A . The number of clauses in ϕ is k , G has one node for each literal in ϕ , and G will have at least $3k$ edges...)

C. Let $A = \{\langle M \rangle : M \text{ is a TM with time complexity } \mathcal{O}(n^2)\}$. Prove that A is not recognizable. (Hint: Mimic a proof from a take-home exam on our course web site.)

D. How many hours have you spent on this exam? (Your answer does not affect your score. I will not read your answer until I have graded your other problems.)