Notes, book, etc. are not allowed.

Except where otherwise noted, you should always justify your answers. Correct answers with no justification may receive little credit. Incorrect or incomplete answers that display insight often receive partial credit.

You have 150 minutes. Good luck.

A. In each part of this problem, there are three valid answers: TRUE, FALSE, and PUNT. If you answer PUNT, then you get half credit. Otherwise, correct answers get full credit and incorrect answers get no credit. Justification is not required or considered in grading.

A.A. The union of a context-free language and a regular language is context-free.

A.B. $\overline{EQ_{TM}} = \{\langle M, N\rangle : L(M) \neq L(N)\}$ is recognizable.

A.C. Let CFL be the set of all context-free languages. Then $CFL \subseteq PSPACE$.

A.D. If A is NP-complete and $A \leq_p B$, then B is NP-complete.

A.E. If a TM uses polynomial space, then it uses at most exponential time.

A.F. Any language in NP is polynomial-time mapping-reducible to TQBF.

A.G. Any NTM of time $\mathcal{O}(f(n))$ has an equivalent TM of time $\mathcal{O}(2^{f(n)})$.

A.H. If A is NP-complete, then the complement $\bar{A}$ is also NP-complete.

B.A. What are the three basic steps in interpreting a statement in a programming language, and what do they have to do with this course's material? In other words, what was the point of our interpreter assignment? (Three sentences should suffice.)

B.B. Explain, using concepts from this course, why interpreters and compilers for real-world programming languages check syntax errors but not semantic errors.

C. Let $A = \{w\#t : w, t \in \{0,1\}^* \text{ and } w \text{ is a substring of } t\} \subseteq \{0,1,\#\}^*$. Prove that A is not context-free.

D. A *useless state* in a TM M is a state that M never enters. Let A be the language of all M that contain a useless state. More precisely, $A = \{\langle M \rangle : \exists q \in Q_M$ such that $\forall w$, M does not enter q on input $w\}$. Prove that A is undecidable. (Hint: In my reduction, I alter a Σ.)

E. In the ISBN-10 standard for identifying published books, an ISBN is a 10-digit string $a_{10}a_9a_8 \ldots a_2a_1$. The first nine digits are base-10. The last digit a_1 is base-11 (with "X" used to denote 10 in base 11). After the first nine digits are chosen, the last digit is chosen to make the equation $(10a_{10} + 9a_9 + 8a_8 + \cdots + 2a_2 + 1a_1)\%11 = 0$ hold. This property facilitates modest error detection (an error in a single digit, or transposition of two digits).

Describe a DFA whose language is the valid ISBN-10 strings. You will not have time to draw the whole DFA. Draw the start state and all arrows out of the start state. Describe in words and sketches how the rest of the DFA should work, including the exact number of nodes.