

This exam begins for you when you open (or peek inside) this packet. It ends at 11:10 AM on Monday 2018 October 29. Between those two times, you may work on the exam as much as you like. Although I do not intend the exam to require more than a couple of hours, you should get started early, in case you want to spend more time. The exam is open-book and open-note:

- You may use all of this course’s materials: the Sipser textbook, your class notes, your old homework, and the materials on our course web site. If you missed a class and want to get some other student’s notes, then do so before either of you begins the exam. You may not share any materials with any other person while you are taking the exam.
- You may cite theorems and examples from class, the assigned textbooks readings, and the assigned homework problems. You do not have to reprove them. On the other hand, you may not cite results that we have not studied without proof.
- You may not consult any other books, papers, Internet sites, etc. You may use a computer for viewing the course web site, running Python programs of your own creation, typing up your answers, and e-mailing with me. If you want to use a computer for other purposes, then check with me first.
- You may not discuss the exam in any way — spoken, written, etc. — with anyone but me, until everyone has handed in the exam. During the exam period you will inevitably see your classmates around campus. Refrain from asking even seemingly innocuous questions such as “Have you started the exam yet?” If a statement or question conveys any information about the exam, then it is not allowed. If it conveys no information, then you have no reason to make it.

Feel free to ask clarifying questions in person or over e-mail. You should certainly ask for clarification if you believe that a problem is mis-stated. Check your e-mail occasionally, in case I send out a correction.

Your solutions should be thorough, self-explanatory, neat, concise, and polished. You might want to work on scratch paper, and then recopy your solutions. Alternatively, you might want to type your solutions. Always show enough work and justification so that a typical classmate could understand your solutions. If you cannot solve a problem, then write a brief summary of the approaches you’ve tried. Partial credit is often awarded. Present your solutions in the order assigned, in a single stapled packet.

Good luck.

**A.** Problems 4.10 and 4.11 are about testing whether a given DFA or PDA has an infinite language. Is the analogous problem for Turing machines decidable? Prove your answer. (I am not asking you to do Problem 4.10 or Problem 4.11.)

Recall that a stack is a data structure that lets you pop items off the top and push items onto the top. A stack is often described as “last-in-first-out”, because the last item you push is the first item that you subsequently pop.

A queue is another kind of data structure, which is much like a stack except that it is first-in-first-out. Think of a queue as a line of well-behaved people facing toward you. You pop (or “dequeue”) from the front of the queue, and you push (or “enqueue”) at the back of the queue. That’s about all there is to say. (If you have never studied queues, then I am happy to answer questions about them by phone or e-mail during the exam. Just give me plenty of notice.)

Recall that a PDA is, to put it briefly, an NFA with a stack. Similarly, for the sake of this upcoming problem, a *Joshutron* is an NFA with a queue. For a more formal definition, re-read the book’s PDA definitions with a queue in mind. For example, the transition function is of the form  $\delta : Q \times (\Sigma \cup \{\epsilon\}) \times (\Gamma \cup \{\epsilon\}) \rightarrow \mathcal{P}(Q \times (\Gamma \cup \{\epsilon\}))$ , where  $\Gamma$  is the queue alphabet.

**B.** How does the expressive power of Joshutrons compare to the expressive power of NFAs, PDAs, and NTMs? For example, does every NTM have an equivalent Joshutron, and vice-versa? Prove your answer.

**C.** Let *DEC* be the set of all Turing machine encodings  $\langle M \rangle$  such that  $M$  is a decider. Is *DEC* decidable? Prove your answer.

**D.** Let  $P$  denote the deterministic polynomial time-complexity class, as usual. Are all regular languages members of  $P$ , or are no regular languages members of  $P$ , or are some members and some not? Prove your answer.

**E.** How many hours have you spent on this exam? (Your answer does not affect your score, but please do supply an honest answer.)

This paragraph has nothing to do with the exam. It is placed here merely to take up space. You are not obligated to read it at all. In middle school, my classmates and I were told to write essays about what it would be like to live in a utopia — a perfect place. We must have discussed the topic beforehand, because I got the impression that everyone was going to argue that living in a utopia would be boring. So my essay argued that it is logically impossible for a perfect place to be boring, because the boringness would be an imperfection. That’s a devastating argument, but it’s not really complicated or subtle, so my essay just kept saying it again and again in different words. The teacher and I eventually agreed that my argument was perfect but boring.