A1.

Parameters: λ.

Set S of values: $\{0, 1, 2, 3, ...\}$.

PDF: $P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$.

Expectation: $E[X] = \lambda$.

A2. [This is Exercise 3.31.] We are told that $X \sim \text{Pois}(\lambda)$ and that $P(X = 0) = 0.1$. Therefore

$$\frac{\lambda^0}{0!} e^{-\lambda} = 0.1,$$

which implies that $\lambda = -\log 0.1$. Then

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - 0.1 - \lambda e^{-\lambda} = 0.9 + 0.1 \cdot \log 0.1.$$

B. [This is Exercise 3.13.] There are $\binom{52}{13}$ bridge hands. Of them,

$$\binom{4}{1}\binom{13}{4}\binom{13}{3}^3$$

follow the $(4, 3, 3, 3)$ pattern, and

$$\binom{4}{1}\binom{3}{1}\binom{13}{4}^2\binom{13}{3}\binom{13}{2}$$

follow the $(4, 4, 3, 2)$ pattern. Instead of computing $P(4, 4, 3, 2)$ and $P(4, 3, 3, 3)$ explicitly, let's just compute their ratio:

$$\begin{aligned}
\frac{P(4,4,3,2)}{P(4,3,3,3)} &= \frac{\binom{4}{1}\binom{3}{1}\binom{13}{4}^2\binom{13}{3}\binom{13}{2}}{\binom{4}{1}\binom{13}{4}\binom{13}{3}^3} \\
&= \frac{\binom{3}{1}\binom{13}{4}\binom{13}{2}}{\binom{13}{3}^2} \\
&= \frac{3 \cdot 3! \cdot 3! \cdot 13^2 \cdot 12^2 \cdot 11 \cdot 10}{4! \cdot 2! \cdot 13^2 \cdot 12^2 \cdot 11^2} \\
&= \frac{3 \cdot 6 \cdot 6 \cdot 10}{24 \cdot 2 \cdot 11} \\
&= 45/22.
\end{aligned}$$

So the $(4, 4, 3, 2)$ pattern is more than twice as probable as the $(4, 3, 3, 3)$ pattern.

C1. First,

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)\cdots(n-k+1)}{k!}.$$

There are k factors in the numerator. When $n \gg k$, all of these factors are close to n. Therefore the fraction is close to $n^k/k!$.

C2. Well,

$$\begin{aligned}
\frac{\binom{r}{k}\binom{b}{n-k}}{\binom{r+b}{n}} &\approx \frac{\frac{r^k}{k!}\frac{b^{n-k}}{(n-k)!}}{\frac{(r+b)^n}{n!}} \\
&= \frac{n!}{k!(n-k)!}\left(\frac{r}{r+b}\right)^k\left(\frac{b}{r+b}\right)^{n-k} \\
&= \frac{n!}{k!(n-k)!}p^k(1-p)^{n-k},
\end{aligned}$$

where $p = r/(r+b)$. This is the density of a random variable $Y \sim \text{Binom}(n,p)$.

C3. Suppose that drawing a red ball is a "success" and drawing a blue ball is a "failure". Then $X \sim \text{Hyper}(r,b,n)$ counts the number of successes when sampling without replacement, while $Y \sim \text{Binom}(n,p)$ counts the number of successes when sampling with replacement. The simplification makes sense when $n \ll r+b$, because there is little difference between sampling with and without replacement, when we are sampling only a small number (n) of objects from a huge pool (size $r+b$).

D. Let G be the event that the defendant is guilty and D the event that the DNA samples match. We wish to find

$$P(G|D) = \frac{P(D|G)P(G)}{P(D|G)P(G) + P(D|G^c)P(G^c)}.$$

We are told that $P(G) = 1/10$ and $P(D|G^c) = 1/100$. Also it is reasonable to assume that $P(D|G) = 1$. Therefore

$$P(G|D) = \frac{1/10}{1/10 + (1/100)\cdot(9/10)} = \frac{100}{109}.$$

[This probability is about 92%. In my opinion, it is not "beyond a reasonable doubt".]

E. [This is essentially Example 3.7 with a twist.] Let X be the amount of money won. Then

$$E[X] = 0 \cdot P(X=0) + 1 \cdot P(X=1) = P(X=1).$$

To compute $P(X=1)$ is to compute the probability of winning a game of craps:

$$\begin{aligned}
P(X=1) &= P(7) + P(11) + \sum_{k\in\{4,5,6,8,9,10\}} P(k)P(k \text{ before } 7) \\
&= P(7) + P(11) + \sum_{k\in\{4,5,6,8,9,10\}} P(k)\frac{P(k)}{P(k)+P(7)} \\
&= P(7) + P(11) + \frac{(3/36)^2}{3/36+6/36} + \frac{(4/36)^2}{4/36+6/36} + \frac{(5/36)^2}{5/36+6/36} \\
&\quad + \frac{(5/36)^2}{5/36+6/36} + \frac{(4/36)^2}{4/36+6/36} + \frac{(3/36)^2}{3/36+6/36}.
\end{aligned}$$

[By the way, the answer is approximately 0.4929.]