A. Suppose that X and Y are jointly distributed with PDF $f(x, y) = c(x^2 + y^3)$, where $1 \leq x \leq 2$ and $1 \leq y \leq 2$.

A1. Compute the normalizing constant c.

A2. Are X and Y independent?

A3. [The old Problem A3 was too hard in its geometry. See the new Problem A below.]

New A. Suppose that X and Y are jointly distributed with PDF $f(x, y) = cx^{-2}y^{-2}$ for $x, y > 1$.

New A1. Find c.

New A2. Are X and Y independent?

New A3. Let $Z = 2X + Y^2$. What is the domain of $f_Z(z)$?

New A4. For a fixed value z, find where the curve $2x + y^2 = z$ hits the lines $x = 1$ and $y = 1$.

New A5. Express $F_Z(z)$ as a double integral over an appropriate region of the x-y-plane.

New A6. Find the PDF of Z.

B. You have three coins. Two of the coins are fair. The other coin is biased, such that it lands heads 3/4 of the time. You pick one of the coins (uniformly randomly) and flip it three times. It lands heads all three times. Given this information, what is the probability that you picked one of the fair coins?

C. Recall that in the game of bridge the 52-card deck is dealt to four players, so that each player has a 13-card hand. What is the probability that a hand contains exactly 9 cards of a single suit?

D. Let $X \sim \text{Norm}(\mu, \sigma^2)$ and $Y = X^3$. Find the density of Y.

E. (This problem comes out of our discussion about the Jeffreys prior distribution for the dispersion of the normal distribution. Don't worry about the domains of the random variables.) Suppose that S has density $f(s) = 1/s$.

E1. Let k be a positive constant and $T = kS$. Show that $f_T(t) = 1/t$. (This is the scale-invariance property of $f(s)$. It says that if T is just S measured in different units, then the prior for T is identical to the prior for s.)

E2. Find the density of $L = \log S$ and the density of $V = S^2$.

F. On each iteration of our ordinary (not necessarily Bayesian) MCMC algorithm, we perturb x_i to a new value x by sampling x from $\text{Norm}(x_i, \sigma^2)$. Sometimes this new x becomes x_{i+1}, and sometimes x_i is repeated as x_{i+1}. Define the *acceptance rate* to be the fraction of MCMC iterations in which x becomes x_{i+1}. What do you think the relationship between σ^2 and the acceptance rate is? If σ^2 is small, then is the acceptance rate small or large? What if σ^2 is large?

G. Let $X \sim \text{Geom}(p)$. From the definitions, show that $E[X] = 1/p$ and $V[X] = (1-p)/p^2$.

H. Recall from our Exam A practice problems that $X \sim \text{NegBin}(r, p)$ counts the number of Bernoulli trials (independent with probability p) required for the rth success. Here, r is any positive integer and $0 \leq p \leq 1$. We found the PDF to be

$$P(X = k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$$

for $k = r, r+1, r+2, \ldots$.
H1. Write an expression for $E[X]$ based on the definition of expectation. Your answer will be a sum, series, or integral. Probably you will not be able to compute it.
H2. Explain how X is a sum of r independent geometric random variables.
H3. Use that insight to compute $E[X]$. Compute $V[X]$ too.

I. [This is Exercise 6.40.] Suppose that X has density $f(x) = (1+x)^{-2}$ for $x > 0$. Show how to use the inverse transform method to simulate X.

J. [This is Exercise 9.29.] Let $X \sim \text{Binom}(m, p)$ and $Y \sim \text{Binom}(n, p)$ be independent. Use MGFs to show that $X + Y \sim \text{Binom}(m+n, p)$.

K. [This is an altered Exercise 8.12.] Suppose that X and Y have joint density $f(x, y) = x + y$ for $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Find $E[X|Y = y]$, $E[X|Y]$, and $E[E[X|Y]]$.

Other topics. With more time, I would have written problems about: central limit theorem, covariance.

Notable things to memorize

- Basic calculus stuff: Limits, including L'Hopital's rule. Differentiation and antidifferentiation. Geometric series and the Taylor series for the exponential function.
- All of our usual discrete distributions: uniform, Bernoulli, binomial, Poisson, geometric. Their PDFs. What situations they commonly model (how to use them in a story problem).
- All of our usual continuous distributions: uniform, exponential, normal. Their PDFs. What situations they commonly model.
- Big definitions: Expectation, variance, covariance, conditional distributions, marginal distributions, PDFs, CDFs, MGFs, etc.
- Big theorems: Bayes' theorem, law of total probability, linearity of expectation, law of total expectation, central limit theorem, etc.

Things not to memorize

- Markov's inequality, Chebyshev's inequality, precise statements of the laws of large numbers. (But you should have an intuitive understanding of the laws of large numbers.)
- Unusual antiderivatives and series. I am not trying to test your calculus skills strenuously.

When in doubt

Memorize it. Officially, you are responsible for all of the material covered in class, the homework, the earlier exams, and the assigned sections of the textbook.