A sequence of equations is a sequence of statements

In math books you frequently see sequences of equations, as in this excerpt:

$$(x-2)(x+3) = x^2 - 2x + 3x - 6 = x^2 + x - 6.$$

Translated into plain text or read aloud, this sentence is

> The quantity x minus 2, times the quantity x plus 3, equals x squared, minus 2 times x, plus 3 times x, minus 6, which equals x squared plus x minus 6.

Notice that the second "=" is pronounced "which equals", so that the sentence is grammatical. When a sentence is written with two "="s like this, it is really making two statements: The first quantity equals the second, and the second equals the third. The two statements could be written separately, as in

$$(x-2)(x+3) = x^2-2x+3x-6.\ x^2-2x+3x-6 = x^2+x-6.$$

However, that is poor writing. The "$x^2 - 2x + 3x - 6$" appears twice, obscuring the logical connection between the two equations and wasting time and space.

In long calculations there may be many equations strung together, as in this snippet:

$$\frac{e^b-e^a}{b-a}(ta+(1-t)b-a) = \frac{e^b-e^a}{b-a}((1-t)b+(t-1)a) = \frac{e^b-e^a}{b-a}((1-t)b-(1-t)a) = \frac{e^b-e^a}{b-a}(1-t)(b-a) = (e^b-e^a)(1-t).$$

Every "=" after the first one is pronounced "which equals", to create one enormous sentence with several clauses. While technically correct, it's difficult to read, compared to this:

$$\begin{aligned}\frac{e^b-e^a}{b-a}(ta+(1-t)b-a) &= \frac{e^b-e^a}{b-a}((1-t)b+(t-1)a)\\ &= \frac{e^b-e^a}{b-a}((1-t)b-(1-t)a)\\ &= \frac{e^b-e^a}{b-a}(1-t)(b-a)\\ &= (e^b-e^a)(1-t).\end{aligned}$$

These two snippets use exactly the same text, and you read/pronounce them identically. The only difference is formatting. In the second snippet, I have aligned the "="s, with just one "=" on each line. It uses a lot of space, but it helps the reader recognize and compare the steps.

Notice that there is still just one period, at the end of the whole sentence. Just as in an English paper, you put a period at the end of every *sentence*, not at the end of every *line*.

Here is an excerpt from a student's homework. By the way, he's solving $x^2 + 5x + 6 = 0$.

$$\begin{aligned} x &= \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 6}}{2} \\ x &= \frac{-5 \pm \sqrt{25 - 24}}{2} \\ x &= \frac{-5 \pm 1}{2} \end{aligned}$$

It's not a disaster (you can understand what he's trying to say), but it's not well-written either. We can fix it up by simply deleting some "x"s and adding a period:

$$\begin{aligned} x &= \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 6}}{2} \\ &= \frac{-5 \pm \sqrt{25 - 24}}{2} \\ &= \frac{-5 \pm 1}{2}. \end{aligned}$$

When writing up your math homework, save yourself writing (and your audience reading) by stringing together equations when you can. For equations longer than one line, align the "="s. The same goes for "<", "≤", ">", and "≥" in inequalities; align them with each other and with any "="s in the sentence.

When you are writing under time pressure (such as on exams or at the chalkboard), it is understandable if your formatting isn't perfect — but still try to write sensible sentences.

Exercises

A. Translate into plain text, keeping no symbols except numeric ones ("1.5", "π", etc.).

$\cos(1.5\pi + 0.5\pi) = \cos(2\pi) = 1.$

B. This sequence of equations is too wide for my paper. Align it.

$$\frac{6x^4 - 2 - 5x^4 + 1}{x - 1} = \frac{x^4 - 1}{x - 1} = \frac{(x-1)(x^3 + x^2 + x + 1)}{x - 1} = x^3 + x^2 + x + 1.$$

C. Read this student's homework. Figure out what he is trying to say. Then rewrite it in a single, well-formatted, grammatical, correct sentence in mathematical notation.

$$\begin{aligned} m &= \frac{(x+h)^3 - x^3}{h}. = \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\ &\frac{3x^2h + 3xh^2 + h^3}{h} \\ &m = 3x^2 + 3xh + h^2 \end{aligned}$$