The arrow "⇒" indicates that one equation implies another

In the previous writing assignment we learned how to present multi-step algebra tasks as strings of equations. However, that doesn't work for all algebra. Consider this snippet:

$$\begin{aligned} x^3 &= 17. \\ x &= \sqrt[3]{17}. \end{aligned}$$

We can't string these equations together. One is talking about x^3 and the other about x, so it would be wrong to equate them. However, we still want to indicate to the reader that the two equations are related, in that the second one follows from the first by simple algebra. This situation arises so frequently that we have a special symbol "⇒" for it:

$$\begin{aligned} && x^3 &= 17 \\ \Rightarrow && x &= \sqrt[3]{17}. \end{aligned}$$

The two equations are now bound together into a single sentence, which is read aloud as

> x cubed equals 17, which implies that x equals the cube root of 17.

The "⇒" is pronounced "which implies that". Intuitively it means, "The following equation is true because of the preceding one and some simple manipulation that isn't worth explaining."

Here is a more complicated example:

$$\begin{aligned} && x^2 + 5x + 6 &= 0 \\ \Rightarrow && x &= \frac{-5 \pm 1}{2}. \end{aligned}$$

The writer is using the quadratic formula and doing several arithmetic steps. Is this calculation so simple that it doesn't merit explanation? That depends on who the intended audience is. For a Math 211 student this is pretty simple, but for a high school student who's just studied algebra it skips far too much. For that kind of reader, we should show more steps:

$$\begin{aligned} && x^2 + 5x + 6 &= 0 \\ \Rightarrow && x &= \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1} \\ && &= \frac{-5 \pm \sqrt{25 - 24}}{2} \\ && &= \frac{-5 \pm 1}{2}. \end{aligned}$$

The "⇒" connects the first equation to the second, which is then continued as a sequence of equations, as in the previous writing assignment. For clarity, all of the "="s are aligned.

How much detail should you show, in your homework and on exams? The simple answer is to *write so that a typical classmate could understand your solution.* When you do, you show enough detail that your grader can determine whether or not *you* understand your solution. When in doubt, show too much rather than too little.

Here is a longer example, taken from a problem in another calculus textbook. The task is to invert the given relationship between Q and t.

$$\begin{array}{rrcl} & Q & = & Q_0\left(1-e^{-t/a}\right) \\ \Rightarrow & Q/Q_0 & = & 1-e^{-t/a} \\ \Rightarrow & e^{-t/a} & = & 1-Q/Q_0 \\ \Rightarrow & -t/a & = & \log(1-Q/Q_0) \\ \Rightarrow & t & = & -a\log(1-Q/Q_0). \end{array}$$

Here we have a sequence of implications, all in one sentence. Read the sentence aloud, pronouncing each "$\Rightarrow$" as "which implies that". Written out in plain text, the sentence would be very long and difficult to read; thank goodness for math notation.

Whenever you write up problem solutions in this course, try to communicate your reasoning in a clear, concise way, including enough detail so that a fellow student could understand your solution. Use clearly formatted equations and implications, to help your reader.

Exercises

A. Translate this sentence into plain text, keeping no symbols except y, x, and 1.

$$\begin{array}{rrcl} & \sqrt{y} & = & x+1 \\ \Rightarrow & y & = & (x+1)^2. \end{array}$$

B. Simone starts with $s^6 = e^{3\ln(t-1)}$. She changes the right-hand side into $e^{\ln(t-1)^3}$ and then into $(t-1)^3$. Then she infers that $t-1$ equals $\sqrt[3]{s^6}$, which equals s^2. Finally she gets $t = s^2+1$. Write up Simone's work as a single sentence in mathematical notation, using equations and implications.

C. Starting from

$$\frac{y+1}{y-1} = 3x,$$

use algebra to solve for y. Show all of your steps in a single sentence in mathematical notation, using equations and implications.