You have 60 minutes.

No notes, books, calculators, computers, etc. are allowed.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

Perform as much algebraic simplification as you can. Do not bother to do non-trivial arithmetic. Mark your final answer clearly.

Good luck.

A. Let $\vec{v} = \langle -2, 3, 1 \rangle$ and $\vec{w} = \langle 2, -1, 2 \rangle$. Calculate these quantities.

$\vec{v} + \vec{w} =$

$7\vec{v} =$

$\vec{v} \cdot \vec{w} =$

$\mathrm{proj}_{\vec{w}}\vec{v} =$

$\vec{v} \times \vec{w} =$

B. A function $f(x, y)$ is said to be *harmonic* if $f_{xx} + f_{yy} = 0$ at all points (x, y). Find numbers b and c such that $f(x, y) = x^3 + bx^2y + cxy^2 + y^3$ is harmonic.

C. Assume that $\vec{r}(t)$ is parametrized by arc length $s = t$. Starting from $\kappa(t) = \dfrac{||\vec{r}\,'(t) \times \vec{r}\,''(t)||}{||\vec{r}\,'(t)||^3}$, derive the usual definition of κ.

D. Again assume that $\vec{r}(t)$ is parametrized by arc length $s = t$. Derive the first Frenet-Serret equation $d\vec{T}/ds = \kappa\vec{N}$.

E1. Let $f(x, y) = \dfrac{x^3 + y^3}{xy^2}$. Compute the limit of f along every line $y = mx$ at the origin, by setting $y = mx$ and computing the limit at $x = 0$.

E2. What can you conclude about $\lim\limits_{(x,y)\to(0,0)} f(x, y)$?

F1. Translate the equation $r^2 = \cos^2\theta - \sin^2\theta$ from polar to Cartesian coordinates (x, y).

F2. Compute $\frac{dy}{dx}$ as a function of x and y.

F3. Find all points on the curve where the tangent line is horizontal.

G1. Sketch a contour plot of $z = f(x, y) = x^2 \sin y$. At three points on your plot, sketch the gradient vector. Select your points such that those three gradients do not all have the same length. Make your sketch capture their relative lengths (short, longer, longest) correctly.

G2. Compute ∇f.

G3. Let $\vec{p} = (1, 2)$ and $\vec{v} = \langle -1/\sqrt{2}, 1/\sqrt{2} \rangle$. Find the derivative of f at $\vec{p}$ in the direction $\vec{v}$.