Suppose that a particle of mass m is sitting on the plane x-y-plane, where $z = 0$. (Mass is measured in kg and distance in m). To raise this particle against Earth's gravitational field to a given altitude $z > 0$ takes work. The amount of work is proportional to the mass m and the lifting distance z. The constant of proportionality is $g = 9.8$ m/s^2.

Now consider a solid, liquid, or gaseous body that occupies a region W of space and has density $\delta(x, y, z)$ (in kg/m^3). Assume that each particle in the body began in the $z = 0$ plane. The total work, to raise all of the particles in the body from $z = 0$ to their current positions, equals $\iiint_W \delta g z \, dV$. This concept can be used to calculate the energy needed to fill a water tower with water, for example. Geologists use such concepts to understand how energy is distributed among geological processes such as mountain building and earthquakes.

In particular, the island of Hawaii is a cone of height of 4200 m above sea level and radius 58000 m. Its density is a constant 3000 kg/m^3. (This is all very approximate. Hawaii is not exactly a cone. Its density is not constant. Most of Hawaii is under sea level, but we're ignoring all of that, even though it is geologically relevant. The acceleration of Earth's gravitational field is not a constant 9.8 m/s^2 over regions as large as Hawaii. In particular, the under-sea part of the island noticeably alters the gravitational field experienced by the over-sea part.)

Exercise A: How much work did it take to raise Hawaii (the above-sea part) from sea level?

In quantum theory, a particle is described by a *wave function* ψ, and the probability of finding the particle in a region D of space is $\iiint_D |\psi|^2 \, dV$. In particular, if we place the nucleus of a hydrogen atom at the origin of our coordinate system, then the wave function for the 1s state of an electron in that atom is $\psi(\rho) = (\pi a^3)^{-1/2} e^{-\rho/a}$, where ρ is the distance to the origin and $a = 5.3 \cdot 10^{-11}$ is a length called the *Bohr radius*.

Exercise B: Compute the probability of finding the electron at a distance of R or less from the nucleus. Give your answer in terms of R and a.

In this problem, we assume that the Earth is a spherical ball of radius $R = 6371$ km. Measurements indicate that the density of the atmosphere drops off exponentially with altitude. To be precise, at an altitude of h km above the Earth's surface, the atmosphere has density $\delta(h) = ae^{-bh}$ kg/km^3, where $a = 1.225 \cdot 10^9$ and $b = 0.13$. There is no clear boundary between the atmosphere and space; the atmosphere just keeps getting thinner and thinner.

Exercise C: Calculate the total mass of Earth's atmosphere, in terms of a and b.