[At the end of class I mentioned that I was going to revise the homework. But let's not. Let's just roll with these homework problems.]

A. [We started this problem in class.] For any two-qbit state $|\chi\rangle$, prove that there exist $\psi_0, \psi_1 \in \mathbb{C}$ and one-qbit states $|\alpha\rangle$, $|\beta\rangle$ such that $|\psi_0|^2 + |\psi_1|^2 = 1$ and

$$|\chi\rangle = \psi_0 \, |0\rangle \, |\beta\rangle + \psi_1 \, |1\rangle \, |\gamma\rangle \, .$$

B. Imagine a two-qbit circuit diagram. The first qbit's wire proceeds uninterrupted across the diagram; that is, nothing happens to the first qbit. The second qbit's wire has a Hadamard gate on it. What is the diagram's overall effect, as a 4×4 unitary matrix?

C. Repeat Exercise B, but with the two wires switched. That is, the top wire has a Hadamard gate, and the bottom wire has nothing on it.

D. The classical NAND gate takes two bits as input and produces one bit as output. Specifically, it maps 00 to 1, 01 to 1, 10 to 1, and 11 to 0. Via our usual trick of turning bit strings into standard basis vectors (such as $01 \leftrightarrow |01\rangle$), we can represent the NAND operation as a non-square matrix. Write out this matrix explicitly.