A. Suppose that the two-qbit state $|\chi\rangle$ enters the CNOT gate unentangled. Show that the state that emerges from the CNOT is entangled — except in rare special cases, which you should enumerate.

B. In class I asserted that a CNOT gate wrapped in four H gates (one on each input and output wire) has matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Show that this matrix is the matrix of an upside-down CNOT (where the control qbit is on the bottom and the data qbit is on the top).

C. Write out all four one-bit classical gates in their two-qbit quantum versions. Describe each answer in two ways. One way is as a 4×4 unitary matrix. The other way is as an expression, made of basic gates such as X, H, and CNOT, combined using matrix multiplication and tensor products. [We have probably done half of this problem in class.]