A. Prove that, if $|\alpha\rangle$ is an n-qbit state and $|\beta\rangle$ is an m-qbit state, then $|\alpha\rangle \otimes |\beta\rangle$ is an $(n+m)$-qbit state. [What must you check?]

B. Verify that the 3-qbit state $|101\rangle = |1\rangle \otimes |0\rangle \otimes |1\rangle$ is indeed the standard basis vector $|101\rangle \in \mathbb{C}^8$. [So this tensor product notation does not conflict with our other notation.]

C. The three-qbit NAND gate should be implementable in terms of the three-qbit AND and a NOT. Explain exactly how, correctly using tensor notation.

D. Here is a classical 7-bit operation, expressed in linear algebra notation:

$$|\alpha\beta\gamma\delta\zeta\eta\theta\rangle \mapsto |\alpha\rangle\,|\beta\rangle\,|\gamma\rangle\,|\delta\rangle\,|\zeta \oplus (\alpha \cdot \gamma) \oplus (\alpha \cdot \beta \cdot \delta) \oplus (\gamma \cdot \beta \cdot \delta)\rangle\,|\eta \oplus \alpha \oplus \gamma \oplus (\beta \cdot \delta)\rangle\,|\theta \oplus \beta \oplus \delta\rangle\,.$$

Is it invertible? What does it do, in English? [And have I made any mistakes in building it?]