A. Section 7.8 Exercise 38.

B. Section 7.8 Exercise 40.

C. Section 7.8 Exercise 55.

D. Suppose that you're investing money in assets such as stocks and bonds. You pay \$$X$ to buy an asset. Sometime later, you sell it and receive \$$Y$. Then the *rate of return* is defined as $R = (Y - X)/X$. For example, if $X = 100$ and $Y = 117$, then $R = 17\%$.

Now suppose that there are n assets with buy prices $X_1, \ldots, X_n$, sell prices $Y_1, \ldots, Y_n$, and rates of return $R_1, \ldots, R_n$. Let w_i be the fraction of your money invested in the ith asset. So $\sum w_i = 1$ and $X_i = w_i X$, where $X = \sum X_i$ is your total investment. Let $Y = \sum Y_i$ be your total selling price and $R = (Y - X)/X$ your overall rate of return.

D.A. Show that $E(R) = \sum w_i E(R_i)$.

For concreteness, suppose that $n = 3$, $w_1 = 25\%$, $w_2 = 43\%$, and $w_3 = 32\%$. Based on the past performance of the assets (which may not be indicative of future performance), you estimate that $E(R_1) = 10\%$, $E(R_2) = 15\%$, and $E(R_3) = 13\%$. You also estimate that $SD(R_1) = 8\%$, $SD(R_2) = 12\%$, and $SD(R_3) = 10\%$.

D.B. What is your expected overall rate of return $E(R)$?

D.C. Assume that the assets have nothing to do with each other, so that their rates of return are uncorrelated. What is the SD of your overall rate of return? How does it compare to the SDs of the individual rates of return? (You will need concrete numbers.)

D.D. Now, do not assume that the rates of return are uncorrelated. How large could the SD of your overall rate of return be? (Hint: Understanding Theorem 7.3.5 might help.)

E. [This problem has nothing to do with covariance. It's about old material.] Commonly, mathematicians rigorously define the concept of *continuous random variable* using a kind of math called measure theory. That's overkill for Math 265. Almost all of our examples or exercises are adequately served by the vague non-definition given in class on Day 14. But this example needs more care.

Consider the following experiment. You flip a coin. If the coin lands heads, then the output of the experiment is 0. If the coin lands tails, then the outcome of the experiment is uniformly random on the interval $[0, 1]$. Let X be the outcome of the experiment.

E.A. What is the support of X?

E.B. Does X have a CDF — meaning a function F such that $F(x) = P(X \leq x)$? If not, why not? If so, then graph it and write an expression for it.

E.C. Does X have a PDF? Again, explain.