You have 70 minutes.

No notes, books, calculators, computers, etc. are allowed.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

Perform as much algebraic simplification as you can. Do not bother to do non-trivial arithmetic unless it is specifically requested. Mark your final answer clearly.

Good luck.

A1. What parameters does the Poisson distribution have? If X is a Poisson-distributed random variable, then what is its set S of possible values? What is its probability (density, mass) function? What is its expectation? (You are not required to show work on Problem A1.)

A2. Suppose that the number of eggs laid by a lizard is a Poisson random variable. The probability that the lizard lays no eggs is 10%. What is the probability that the lizard lays at least two eggs?

B. In the card game known as bridge, the 52-card deck is dealt to four players, so that each player has 13 cards. Many bridge players believe that the most probable distribution of suits in a bridge hand is $(4, 3, 3, 3)$ (four cards of one suit and three cards of each of the other suits). Show that, in fact, the $(4, 4, 3, 2)$ distribution is more probable.

C. The *hypergeometric distribution* $\text{Hyper}(r, b, n)$ is defined by $P(X = k) = \frac{\binom{r}{k}\binom{b}{n-k}}{\binom{r+b}{n}}$ for k such that $\max(0, n - b) \leq k \leq \min(n, r)$. If an urn contains r red balls and b blue balls, we sample n balls without replacement, and X is the number of red balls sampled, then $X \sim \text{Hyper}(r, b, n)$.

C1. Explain this approximation: $\binom{n}{k} \approx \frac{n^k}{k!}$ when $k \ll n$.

C2. Use the approximation to simplify the hypergeometric distribution to another familiar distribution.

C3. Explain in English why that simplification makes sense when $n \ll r + b$.

D. A jury must determine whether a defendant is innocent or guilty. After hearing preliminary, circumstantial evidence, the jury is 10% certain that the defendant is guilty. Then a forensics expert testifies that the defendant's DNA matches DNA found at the crime scene. She also estimates that the probability of the crime scene DNA's having come from someone other than the defendant is 1%. Given this new information, how certain should the jury be of the defendant's guilt? Arithmetically simplify your answer to a ratio of two integers with no common factors.

E. When playing the game of *craps*, you roll two dice and add them. If the sum is 7 or 11, then you immediately win \$1. If the sum is 2, 3, or 12, then you immediately lose (you win \$0). If the sum is any other number, then that number is called your *point*. In that case, you roll repeatedly until you get either 7, in which case you lose, or your point, in which case you win \$1. How much do you expect to win in a game of craps?