You are allowed one one-sided sheet of notes, per my earlier instructions. You are not allowed books, calculators, computers, etc.

Except on TRUE-FALSE questions, show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

I sometimes ask you to derive/show/prove facts that we have studied. In such a problem, you may not simply cite the fact as true. That would trivialize the problem. You should instead derive the fact from more basic concepts and theorems. Ask me for clarification if necessary.

Define any notation that you introduce. For example, if you write "$P(A)$", but neither I nor you have defined an event A, then that writing is not good.

Perform as much algebraic simplification as you can. Do not bother to do non-trivial arithmetic unless it is specifically requested. Mark your final answer clearly.

You have 150 minutes. Good luck. :)

A.A. In class we discussed a Bayesian MCMC simulation, in which we deduced the parameter values μ and σ^2 for a normal distribution, based on a data set sampled from that distribution. Rather than simulate $\langle \mu, \sigma^2 \rangle$ or $\langle \mu, \sigma \rangle$, we simulated $\langle \mu, \log \sigma \rangle$. Why? (Hint: Think about how new values of the parameters are produced by the Metropolis-Hastings algorithm.)

A.B. Let $f_\Sigma(\sigma) = 1/\sigma$. (This is the Jeffreys prior for σ. Don't worry about its support, whether it integrates to 1, etc.) Let $L = \log \Sigma$. Derive $f_L(\ell)$ explicitly from $f_\Sigma(\sigma)$.

B.A. Let X and Y be continuous random variables with joint distribution $f_{X,Y}(x, y) = e^{-y}$, supported where $0 < x < y < \infty$. Find the marginal distribution of Y, including its support.

B.B. Find the conditional expectation $E(X|Y = y)$.

B.C. Find a simple expression for the random variable $E(X|Y)$, as a function of whatever random variables, of which it's a function.

C.A. Let c be constant and $Y = X + c$. From the definition of the MGF, derive a simple expression for $m_Y(t)$ in terms of $m_X(t)$ and c.

C.B. If $W \sim \text{Norm}(\mu, \sigma^2)$, then what is $m_W(t)$?

C.C. Let $T \sim \text{Binom}(n, p)$. Derive $m_T(t)$ from the underlying Bernoulli distribution.

C.D. Use the central limit theorem to find an approximation for $m_T(t)$ when n is large.

D. Recall that if X and Y are independent continuous random variables and $T = X + Y$, then $f_T(t) = \int_{-\infty}^{\infty} f_Y(t-x) f_X(x)\, dx$. Derive the analogous expression for $f_W(w)$, where $W = XY$. To make the solution simpler, please assume that X is supported on the positive real numbers.

E. On these TRUE-FALSE questions there are four valid answers. If the correct answer is TRUE, then TRUE earns 3 points, TRUISH earns 2 points, FALSISH earns 1 point, and FALSE earns 0 points. If the correct answer is FALSE, then these point values are of course reversed. Do not write just T or F; write your answer completely and clearly. No explanation is needed.

E.A. If $X \sim \text{Unif}(0,1)$, then $f_X(-1)$ is undefined.

E.B. If A and B are independent events such that $P(A|B)$ is defined, then $P(A|B) = P(B)$.

E.C. In a Poisson process with rate λ, the number of arrivals in a time interval of length t is an $\text{Expo}(\lambda t)$ random variable.

E.D. If X is the result of a die roll and $Y = 7 - X$, then X and Y have the same distribution.

E.E. If X is the result of a die roll and $Y = 7 - X$, then X^2 and XY have the same distribution.

F. Three companies manufacture air bags for cars. The first company, Acme, supplies 50% of the market, and 0.6% of its air bags are defective. For the second company, Bagatronic, these numbers are 35% and 0.2%. For Carface these numbers are 15% and 0.7%.

F.A. What is the probability that a uniformly randomly selected air bag is defective?

F.B. All six air bags in your car were made by one company (but you don't know which company). What is the probability that none of them are defective?

F.C. The steering wheel air bag in your car is defective. What's the probability that it was made by Acme?

G. Suppose that a life insurance customer has fixed probability p of dying each year. (This assumption is unrealistic, but it keeps the problem simple.) Each year that he survives he pays \$100 to the insurance company. How much should he expect to pay over the life of the policy?