

You have 70 minutes.

No notes, books, calculators, computers, etc. are allowed.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

Do not write “+” to mean “and”. Define any notation that you introduce. For example, if you write “ $P(A)$ ”, but neither I nor you have defined an event A , then that writing is not good.

Perform as much algebraic simplification as you can. Do not bother to do non-trivial arithmetic unless it is specifically requested. Mark your final answer clearly.

Good luck.

A. If X and Y are uncorrelated, then must it be true that $V(X + Y) = V(X) + V(Y)$? Explain.

B. Let $X \sim \text{Unif}(0, 1)$. Let F be any CDF that is one-to-one and continuous. Let $Y = F^{-1}(X)$.

B.A. What is the CDF of Y ?

B.B. Suppose that Y is the score for a randomly chosen student on a standardized exam with many questions and many students. What then is the meaning of the random variable $F(Y)$?

C. *Roulette* is a gambling game played in casinos. All that you must know about it is that, if X is the casino's earnings on a single run of the game, then $P(X = 1) = 20/38$ and $P(X = -1) = 18/38$. Let S_n be the casino's total earnings from n independent runs of the game.

C.A. What is $E(X)$?

C.B. What is $SD(X)$?

C.C. What is $E(S_n)$?

C.D. What is $SD(S_n)$?

D. In genetics, a phenomenon known as *crossover* can occur at any point along a chromosome. Geneticists are particularly interested in the case where there is an odd number of crossovers in a chromosome segment, because it means that a *recombination* has occurred in that segment.

It is common to model crossovers using a Poisson process with rate λ . To clarify: We've talked about using Poisson processes to model occurrences in an interval $[0, t]$ of time, but they can just as well be used to model occurrences in an interval $[0, \ell]$ of length.

D.A. Let X be the number of crossovers in $[0, \ell]$. What is the distribution of X ? Be sure to specify its parameters in terms of λ , ℓ , and any other necessary quantities.

D.B. What is the probability of a recombination in $[0, \ell]$? Leave your answer in non-closed form: a sum, series, or integral.

D.C. Evaluate that probability in closed form. Hint: First evaluate $P(X \text{ is even}) - P(X \text{ is odd})$.

E. Recall that a Rayleigh-distributed X has PDF $f_X(x) = xe^{-x^2/2}$ on support $(0, \infty)$. Suppose that I choose a Rayleigh X and then you choose $Y \sim \text{Norm}(X, 1)$.

E.A. What is the joint PDF of X and Y ?

E.B. What is the marginal PDF of Y ? You may leave your answer in non-closed form: a sum, series, or integral.

E.C. Are X and Y independent? Explain.